Comparative Analysis of Compositional Two-Phase Flow Modeling in Heterogeneous Media Between the Discrete Event Simulation Method Coupled to a Split Node Formulation and Classical Timestep-Driven Approaches

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Outline

- Introduction to Discrete Event Simulations
- Discrete Event Simulations for Hybrid Finite Element Finite Volume Method
- Two Phase Flow Simulations: TDS vs. DES
- Material interfaces in Hybrid Finite Element Finite Volume Method
- Conclusions

Motivation

Simulating multiphase flow imposes certain restrictions on the time step length due to the complexity of the reservoir and related emergent behaviour .

The asynchronous time step methodology yields a speed up, particularly under the following conditions:

- High Mesh Resolution in specific regions (e.g. Small mesh near the wells, fractures, faults)
- High Permeability Contrasts (e.g. existence of high permeability zones such as fractures)
- Highly Non-linear Problems (e.g. non-linear two phase models such as Brooks Corey, two-phase gravitational effects)

Adaptime time-refinement approach

In adaptive time-refinement the solution updates locally in each cell by local time-increment. To simplify and to increase the efficient of synchronization procedure the time-steps usually taken as fractions of global time $\frac{\Delta t}{2^n}$.

This approach is not increasing the accuracy of calculations, but at the same time it doesn't reduce the approximation order of numerical scheme. Thus the adaptive time-refinement will be more efficient than the general global-time stepping.

Let us consider the 1D heat equation and the numerical scheme of second order of approximation:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\kappa(t, x, T)\frac{\partial T}{\partial x}\right) \quad (1)$$

$$\frac{T_m^{n+1} - T_m^n}{\tau} = \frac{\kappa}{\Delta x^2}\left(T_{m-1}^n - 2T_m^n + T_{m+1}^n\right) \quad (2)$$

Adaptime time-refinement approach

Let as assume that there are several regions $i = 1..m$, with different spatial sizes of cells, for instance as $\Delta x_i = 2^{i-1}\Delta x_1$, where Δx_1 the smallest size of cell.

Each group of cells will be defined by numbers n_i and $p_i = \frac{n_i}{N}$, where n_i number of cells in each region, p_i percent of cells from each group corresponding to the total number. Let us additionally assume that $\kappa = 1$.

Then we will get $\Delta t_i = \frac{\Delta x_i^2}{2} = 2^{2(i-1)}\frac{\Delta x_1^2}{2} = 2^{2(i-1)}\Delta t_1$, where $\Delta t_1 = \Delta t_{min}$.

Let also τ_{avg} be the average time for calculation procedures for each cell during one time-step.

Adaptime time-refinement approach

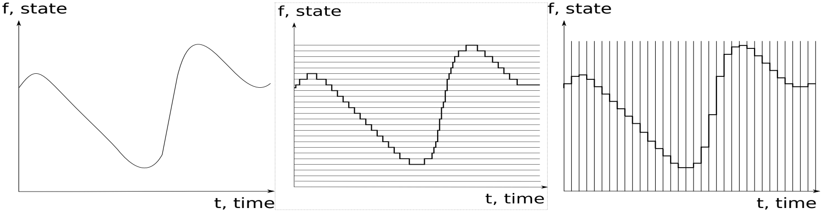
We can now estimate the time required for calculations in global time-stepping and self-adaptive local time-stepping approaches.

$$r = \frac{t_{GTS}}{t_{LTS}} = \frac{\tau_{avg} \frac{t_{end}}{\Delta t_{min}} N}{\tau_{avg} \frac{t_{end}}{\Delta t_{min}} \sum_{i=1}^{m} \frac{n_i}{2^{2(i-1)}}} = \frac{1}{\sum_{i=1}^{m} \frac{p_i}{2^{2(i-1)}}} \tag{3}$$

So for instance if we will have the two groups of cells with corresponding percentages $p_1 = 0.1, p_2 = 0.9$ then the ration of GTS time to LTS time will be $r \simeq 3$.

Timestep Driven Simulations and Discrete Event Simulations

The way to distinguish Discrete Event Simulation (DES) from Timestep Driven Simulation (TDS) can be found in the nature of the state space.



TDS : $(X_d, T_d) \longrightarrow Q$ – *is the mapping from discrete time – discrete space to continuous state space*

DES : $(X_d, T) \longrightarrow Q_d$– *is the mapping from continuous time – discrete space to discrete state space (Quantized State System)*

Discrete Event Simulations for ODE's

Taking the Cauchy ODE problem as an example, there will be a need to integrate right hand side function $f(x,t)$:

$$\dot{x}(t) = f\left(x(t),t\right),\; x(t_0) = x_0$$
$$x(t_i + \Delta t_i) = x(t_i) + \int\limits_{t_i}^{t_i+\Delta t_i} f\left(x(\tau),\tau\right) d\tau$$

DES and TDS approaches will view on these integrals differently:

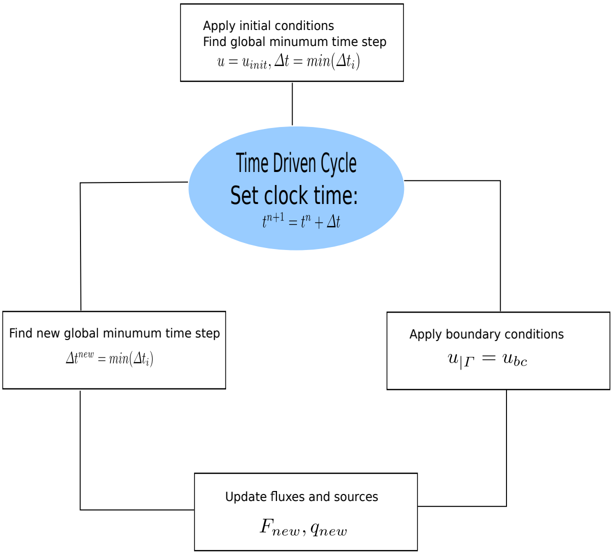
$$TDS: \int\limits_{t_i}^{t_i+\Delta t_i} f\left(x(\tau),\tau\right) d\tau\text{: Riemann Integral}$$
$$DES: \int\limits_{t_i}^{t_i+\Delta t_i} f\left(x(\tau),\tau\right) d\tau\text{: Lebesgue Integral}$$

Bringing DES to the Finite Element-Finite Volume Framework

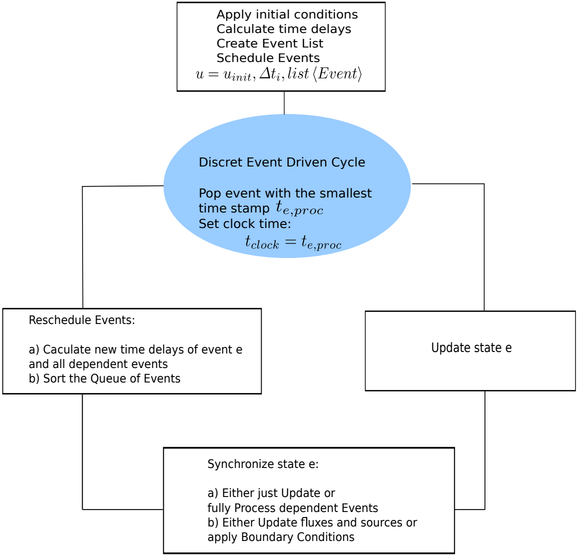
Discrete event simulations for solving partial differential equations were applied in conjunction with FD's, FV's and PIC approaches by now [Omelchenko2006].
But it can also be used together with other discretization methods and it seems that there is no significant restrictions for that.
From the algorithmic point of view, to apply DES to the FEFV framework each Finite Volume must be associated with an Event Process, which will have the following main attributes:

- Scheduled Process Time
- Target Scalar Change (value constrained by a state change that is significant for corresponding Finite Volume)

Time Driven Simulation Cycle



Discrete Event Simulation Cycle



Discrete Event Simulation in FEFV framework

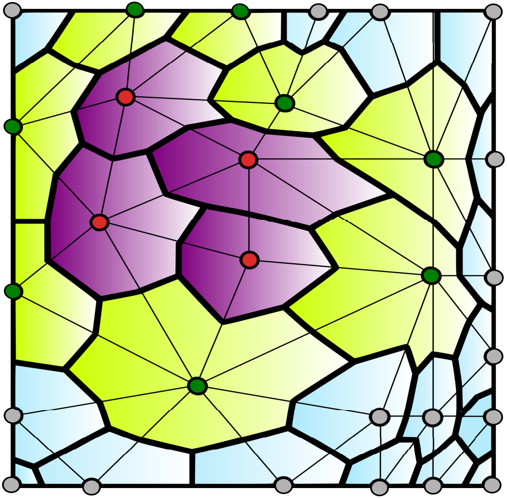
Thus the DES algorithm in general and also specifically in FEFV cell will consist on the following steps:

- Initialize all the events and schedule them (predict the delays of each event)
- Process the event with smallest process time (Update, Recalculate the Fluxes, Synchronize and Reschedule this Event).
  Synchronization: trigger the chain process of updating and processing the neighbour events until it will reach the ones which shouldn't be process, because they didn't reach the target scalar change.
  Reschedule: reschedule the event based on the new values of fluxes and process time, and resort the queue.
- Repeat the above steps until the queue is empty

Discrete Event Simulation in FEFV framework

To increase the efficiency of the flux calculations in FEFV, we also introduce the Event attribute called *Rate Status*. Based on the current status of a nodal variable (associated with FEFV cell) the above algorithm can thus be applied to an *Element Centred Event*.
This gives the possibility to visit the elements and provide the synchronization and calculation processes immediately for all the FV sectors of an FE cell, without repetition.
Such approach can also extend application of DES to Finite Element Framework.

Element Centered DES



Rate of FV variable should be updated, and FV process should be resheduled

Rate of FV variable should be updated, but FV process should not be resheduled

Rate and status of FV variable stays the same

DES with Preemptive-Event-Processing

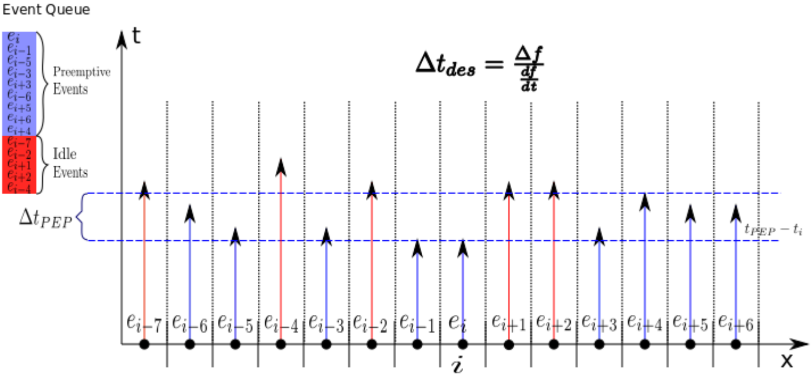
The standart DES approach can be improved by Preemptive-Event-Processing technique introduced by [Omelchenko2006]. It had the intention to bring DES into parallel world.

The main idea of Preemptive-Event-Processing is to group all the event with small difference in expected time of processing δt_{PEP} and to do the synchronization automatically for this group, without waiting for the call of confluent function from each of this events.

For handling this group of events the additional construction PEP Stack is used in order to do the internal sorting process in small group instead of sorting the whole Event Queue.

The control parameter δt_{PEP} can be used either as a global or local condition on state variable change.

DES with Preemptive-Event-Processing



Slightly compressible immisccible two phase flow

Pressure equation:

$$\begin{aligned}\phi c_t \frac{\partial p_w}{\partial t} &- \nabla \cdot \lambda_t \mathbf{K} \nabla p_w + \nabla \cdot \left(\rho_w^{-1} \mathbf{v}_w \nabla \rho_w + \rho_n^{-1} \mathbf{v}_n \nabla \rho_n \right) = \\ &= q_t + \nabla \cdot \lambda_n \mathbf{K} \nabla p_c - \nabla \cdot \mathbf{K} \left(\lambda_w \rho_w + \lambda_n \rho_n \right) \mathbf{g}\end{aligned}$$

Saturation equation:

$$\begin{aligned}\frac{\partial \left(\phi \rho_n S_n \right)}{\partial t} - \nabla \cdot \left(\rho_n \lambda_n \mathbf{K} \left(\nabla p_w - \rho_n \mathbf{g} \right) \right) - \nabla \cdot \left(\lambda_n \rho_n \mathbf{K} \nabla p_c \right) &= \rho_n q_n, \\ &\textit{or} \\ \frac{\partial \left(\phi \rho_n S_n \right)}{\partial t} + \nabla \cdot \left(\rho_n f_n \left(\mathbf{v}_t - \lambda_w \mathbf{K} \nabla p_c - \lambda_w \left(\rho_w - \rho_n \right) \mathbf{g} \right) \right) &= \rho_n q_n\end{aligned}$$

Incompressible immisccible two phase flow

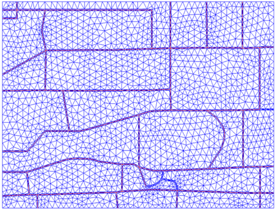
Pressure equation:

$$\int_\Omega \lambda_t \mathbf{K} \nabla p_w \cdot \nabla N d\Omega = \int_\Omega q_t N d\Omega +$$
$$+ \int_\Omega \mathbf{K} \left(\lambda_w \rho_w + \lambda_n \rho_n \right) \mathbf{g} \cdot \nabla N d\Omega - \int_\Omega \lambda_n \mathbf{K} \nabla p_c \cdot \nabla N d\Omega$$

Saturation equation:

$$\int\limits_\Omega W \frac{\partial}{\partial t} \left(\phi S_n \right) d\Omega - \int\limits_\Omega W q_n d\Omega$$
$$- \int\limits_{\partial\Omega} W \cdot \left(\frac{k_{rn}}{\mu_n} \mathbf{K} (\nabla p_w + \nabla p_c - \rho_n \mathbf{g}) \right) \cdot \mathbf{n} \, d\Gamma = 0$$

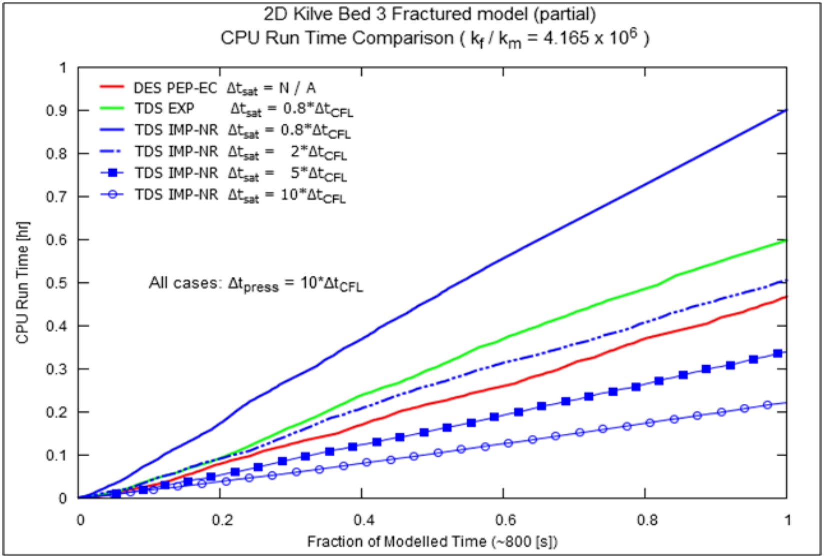
Kilve Bed 3 Section 2D Results



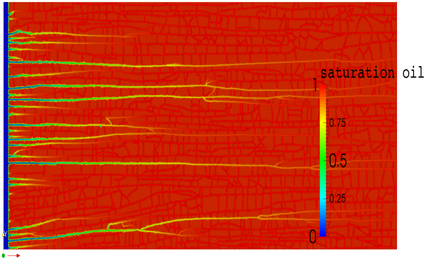
Simulation characteristics:

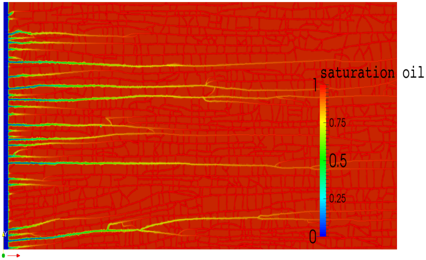
- $K_m = 2.0e - 14$
- Incompressible two phase flow with gravity effects
- Brooks Corey rel. perm. model
- Injector(nw) well at low left corner, Producer at upper right corner
- $N_{elmts} = 5779$, $\frac{h_{min}}{h_{max}} = 0.04$
- Dimensions: 1.88 x 1.25 m
- Injection rate $= 5.0 \cdot 10^{-5} m^3 s^{-1} (CO_2)$
- $\rho_w = 1045 kg \cdot m^{-3}, \rho_{nw} = 479 kg \cdot m^{-3}$

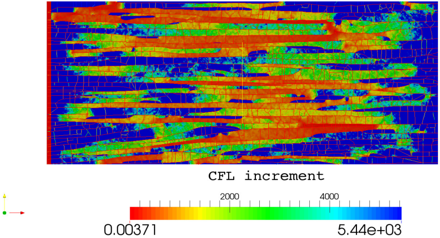
Kilve Bed 3 Section 2D Results



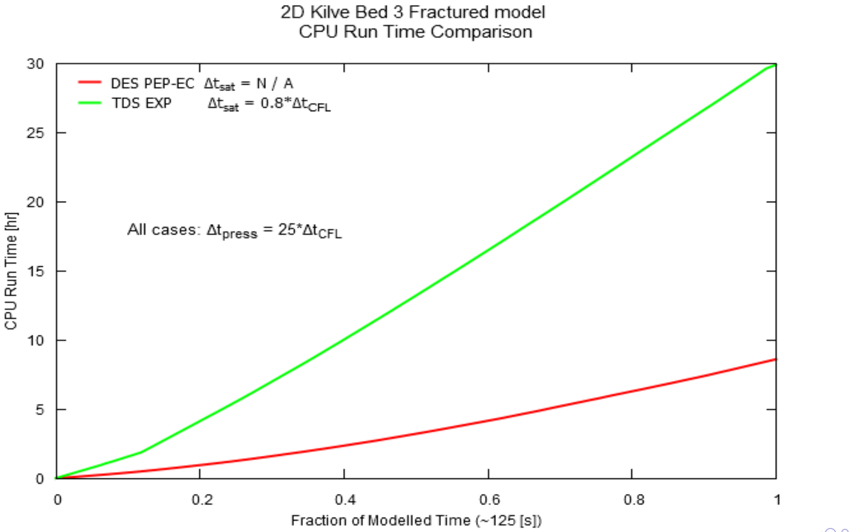
Kilve Bed 3 2D results: DES vs IMPES, saturation plots







Kilve Bed 3 2D perfomance results: DES vs EXP, runtime plots



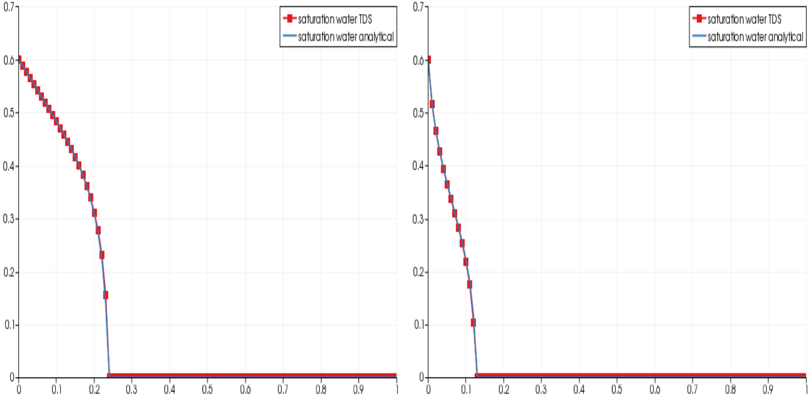
Simulations in heterogeneous media

For producing an accurate results on material interfaces with FEFV method one would require to apply special conditions and discontinuous solution (saturation) across the interface.

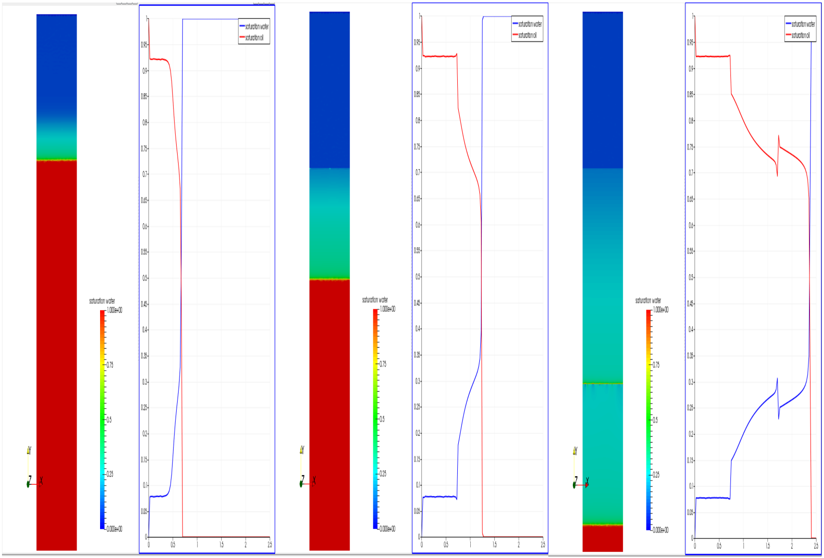
Thus the extended capillary pressure condition was applied in this work on material interfaces [Duijn95].

What might be distinctive in our approach is that the explicit splitting of the nodes was enforced on the mesh level.

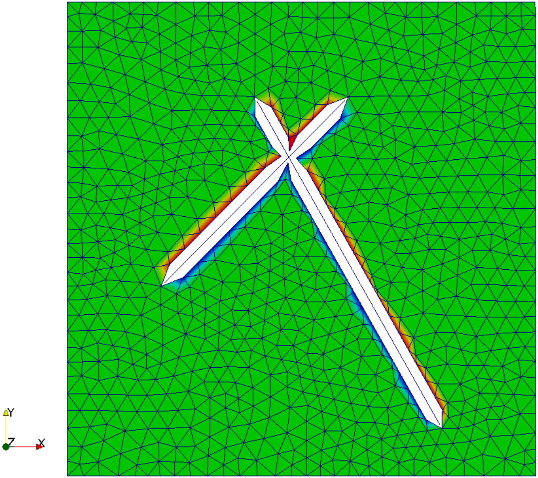
McWhorter Solution: Co-current and Counter-current flow



Three Zones of different materials



SplitNode approach



SplitNode approach

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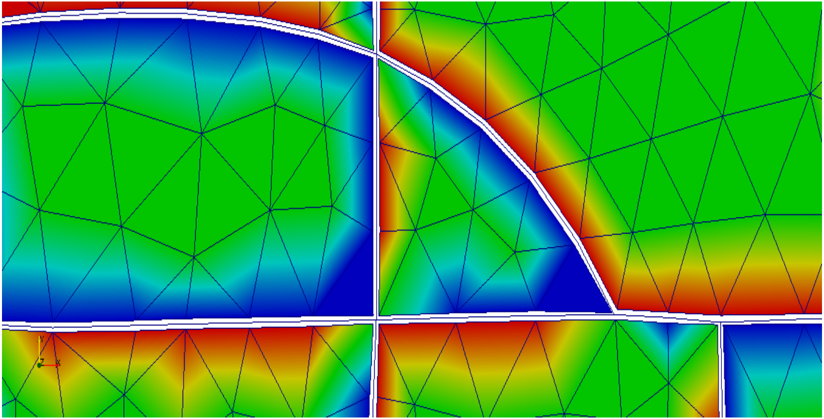

Conclusions and Future Work

- Discrete event simulation is a sufficiently general and flexible approach intended for multi scale type of problems, which have significant variations of characteristic times of different processes.
- The efficiency of DES is dependent on the amount of activity which occurs in the simulation with respect to the total amount of possible events. In general, the lower this ratio is, the better the performance of DES.
- Future work includes the implementation and study of the DES approach for compositional compressible flow in fractured media with application to CO_2 migration in saline aquifers

References

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