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THEMA DER MASTERARBEIT:

Stochastic approach to address and measure grade uncertainty in mining operation for decision making in short term planning

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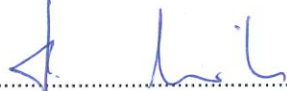
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Declaration of Authorship

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements.

This applies also to all figures, drawings, maps and images included in the thesis.

Freiberg, date

Name

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Abstract

The grade uncertainty is one of the critical aspects that could determine the viability of a mining operation, especially in underground mines. The mining planning uses as input the geological block model usually estimated by the kriging technique, which is the most widely method used to develop the block model. Nevertheless, the smoothing effect generates deviations in ore grade reserves, which produce deviations between planned and executed ore grades. The present work will explain a suitable proposal to overcome the grade uncertainty in mining operations and define a short-term production planning using geostatistical simulation as well as a metaheuristic approach.

The first chapter explains the main problems generated by geological uncertainty in an underground mine operation as well as the proposed solution.

The second chapter defines the general concepts for the stochastic mine planning required to be able to understand the different theories used to estimate, simulate and optimize the block models.

The third chapter exemplifies the methodology followed to be able to develop a method to generate a short-term production plan considering geological uncertainty. Furthermore, will be detailed the new heuristic approach developed to be able to define a mining extraction scheduling in a short time for the "room and pillar" mining method.

The fourth chapter presents the implementation and execution of the project itself in a controlled environment, developing, assessing, valuating and classifying a set of different equiprobable block models, to subsequently develop the short-term production program. It was shown that the kriging estimate had a "high" geological risk and a "moderate" economic potential, so it was chosen among 50 realizations, to work with simulation 19, which had a "low" geological risk and a "high" economic potential.

The fifth and sixth chapters refer to the discussion of the results obtained in each step and the conclusions achieved.

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1 Introduction

Mining planning is an activity that consists of establishing the form (mining method) and sequence (scheduling) of mineral extraction as effective and economical as possible, using as input the geological block model.

Considering that the geological uncertainty generates deviations in the ore reserve estimate, which, according to Butler (1994), is the most important parameter to evaluate in mining projects, it is extremely important to measure and evaluate this uncertainty.

In the present work, a suitable proposal will be explained to overcome the grade uncertainty in mining operations using the conditional simulation and developing a short-term planning scheduling in an appropriate amount of time through the genetic algorithm.

1.1 Definition of the Problem

Mining planning focus on developing a production sequence in which the in-situ ore variability is well addressed, having as an outcome homogeneous products (Benndorf 2020). The fulfilment of this homogeneity can be threatened by geological uncertainty, which can cause a series of problems that condition negatively affect the functioning of the mining operation. These problems are manifested mainly in the following aspects:

- Mistrust in the block model
- Mistrust in mine planning
- Increase in operational improvisation to meet the daily requirement of the processing plant.

The direct effects of not meeting the estimated grades are complex. Having lower grades will cause higher tonnage to be mined, often called unplanned production, which will lead to a reduction of the annual production, an increase of unit operating costs, and reducing the LOM (Butler 1994).

Long-term planning uses the block model as a baseline to be able to develop the reserves calculation and thereby define an optimal mining that allows obtaining the highest possible NPV. This plan is implemented by short- and medium-term planning, which is responsible for guiding the operation to achieve the defined objectives, which are, mainly, to accomplish with the planned tonnage and ore grade. Therefore, it is of paramount importance to evaluate and control the grade variability, aiming to avoid significant plan variations (Benndorf 2020).

Not meeting the expected reserves grade is not an uncommon problem. Baker and Giacomo (2001) collect the information from the deviation between estimated and executed reserves grade, comparing a period of 9 to 12 months in 48 mining projects in Australasia, revealing that 60% of these projects were overestimated or underestimated with a minimum deviation of 15%, as is detailed in Figure 1.

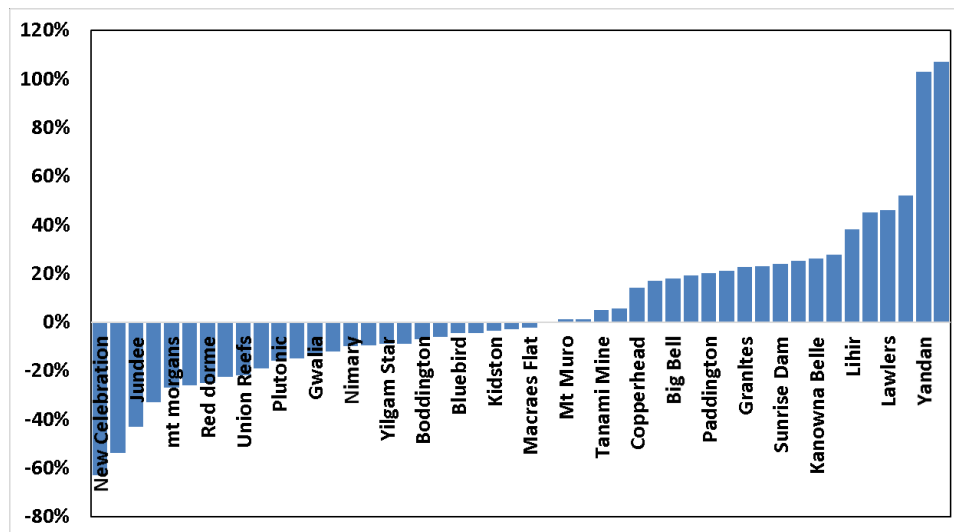


Figure 1: Reserve Grade Deviation (Baker and Giacomo 2001)

There are various sources of information indicating that several mines around the world have failed due to not meeting ore grade expectations (Appleyard 2001).

Focusing on underground operations, because of the different constraints and challenges within the operation that limit the flexibility to modify the infrastructure, the design and planning activities must be highly detailed and precise. Unfavorable conditions in underground mines are intensified when there are especially significant deviations in the expected ore grade (Nelson 2011).

1.2 Proposed Solution

Currently, the most common technique used in the world to estimate ore resources and develop the block model is the Kriging method (Sotoudeh et al. 2019). This technique is considered the Best Linear Unbiased Estimator (BLUE). However, it only generates a single and deterministic estimation, which is not adequate for rational decision making (Rendu 2002).

Consequently, the optimization is developed, and with it, the mining plans. However, the smoothing effect, generated by the Kriging technique, affects the quality of the planning and scheduling especially in stopes with an average grade close to the cut-off, which produces problems in daily operation and sensible deviations in short-term planning.

Once the limitations of the kriging method are understood, it is possible to address and quantify the geological risk before the exploitation using the Conditional Simulation method, specifically using the Sequential Gaussian Simulation technique. One of the advantages of conditional simulations is that it provides a set of equiprobable models called realizations that keep the spatial variability of the original data set and reproduces the histogram as well (Abzalov 2016).

These properties are crucial for proper planning because knowing that the block model is the core of the whole mining planning in all its phases, being able to manage the grade uncertainty will be highly beneficial to the operation itself, since it will reduce the technical risk generated by deviations in the expected ore grade and tonnage (Dimitrakopoulos et al. 2002).

To define the sequence of extraction block by block, is necessary to develop the mining scheduling. However, developing this in a manual and traditional way with several block models is nor efficient nor productive, therefore a new optimization approach is proposed using as a baseline the heuristic approach, the genetic algorithm method.

In general, the genetic algorithm method is a stochastic technique that provides equiprobable solutions in an acceptable period of time. Developing and conditioning this method in the mining scenario, it will allow developing mining plans in a short amount of time. Depending on the mining method, the cut-off grade and the values to maximize or minimize it is possible to develop a logical sequence of extraction block by block.

Once the mining sequence is obtained for each of the simulated block models, these are evaluated by assigning them a valuation, related to the Gaussian properties, considering the economic potential (financial expectation of NPV) and the geological risk (metallic content).

The valuation will allow to classify each simulated and estimated block model into a risk matrix in which it will be possible to visualize the risk and potential and develop a risk assessment for each block model using their respective valuations. Applying all this information it will be possible to select the one with lower geological risk and higher economical value.

The short-term mining plan is developed using the genetic algorithm in the block model selected which will provide us with a short-term production plan considering the geological risk, which will allow us to have a more reliable and feasible plan.

1.3 Research Question

- How to contemplate the geological grade uncertainty inside of the mining planning in order to reduce and manage deviations in short-term planning.
- How do deterministic and stochastic block models compare.
- How to select a mining plan most likely to be achieved meeting the production targets.
- How to develop different mining scheduling for different block models without high time consumption.
- How to classify a block model based on geological risk and economic potential.

1.4 Objectives

1.4.1 General Objectives

Develop a short-term mining plan that considers the uncertainty in the ore grade and, at the same time, can present outcomes in a brief period of time.

1.4.2 Specific Objectives

Develop stochastic model simulations using Sequential Gaussian Simulation of an extraction level of the deposit considered in this study case.

Development of a new approach to optimize the mining extraction scheduling using as a baseline the Genetic Algorithm theory.

Test the optimization algorithm developed in a controllable environment and validate the outcomes of the mining scheduling.

Evaluation and classification of geological risks and economic potential of medium-term mining plans.

Development of short-term production mining plan using as input the best medium-term model.

2 General Concepts in Stochastic Mine Planning

This chapter will explain the basic concepts necessary to understand the process of estimation, simulation and optimization developed in this work. This section is divided into 3 parts. The first refers to the kriging estimation part, in which the formulas and concepts behind the development of this technique will be developed, especially delving into ordinary kriging, which was the chosen method to develop the estimates of grades in the deposit in study. The second part will cover the topic of stochastic simulation, mainly explaining the method and execution procedure of the sequential Gaussian simulation (SGSIM). Finally, the optimization process will be presented, in which its objectives and definitions will be expressed, as well as the steps to be able to develop the Genetic Algorithm (GA) method, which will be used to obtain mining sequences quickly to be able to develop the risk assessment.

2.1 Estimation

In general, estimation refers to the process of inferring unknown values. From a geostatistical point of view, is a concept used to predict a realization of a random function at an unsampled location (Olea 1991).

Considering that the precision of the estimates depends on different factors such as the number, quality and position of the samples, the distance between the samples and spatial continuity of the variable, the kriging method is suitable to develop the estimation since it considers all these factors in its process (Armstrong 1998).

2.1.1 Spatial Estimation: Kriging

Kriging is a local estimation technique that gives the linear unbiased estimator of a determined known characteristic in which it minimizes the variance. This technique is considered as an exact interpolator because is an estimation method that always returns the sample values as estimated values at sample points (Armstrong 1998). The Kriging technique allows obtaining estimated grades of a point by processing the weighted average of known samples grades (Matheron 1963).

All of the different types of Kriging estimators are derivations of the following linear regression (Goovaerts 1997a).

$$Z^*(u) - m(u) = \sum_{\alpha=1}^{n(u)} \lambda_{\alpha}(u) (Z(u_{\alpha}) - m(u_{\alpha})) \quad (1)$$

Where:

- $Z^*(u)$: Linear estimator
- $m(u)$: Expectation of $Z(u)$
- $m(u_{\alpha})$: Expectation of $Z(u_{\alpha})$
- λ_{α} : Assigned weights of $Z(u_{\alpha})$ data
- $n(u)$: number of data points in the local neighborhood

Knowing that kriging must comply with the unbiasedness requirement, which means that the error must be zero, considering that error is the average difference between estimated values and real values. In addition, it must also have a minimum estimation variance. (David 1977).

Unbiasedness:

$$E\{Z^*(u) - Z(u)\} = 0 \quad (2)$$

Minimum estimation variance:

$$\sigma_E^2(u) = Var\{Z^*(u) - Z(u)\} \quad (3)$$

For these conditions, Kriging is broadly considered as the Best Linear Unbiased Estimator - BLUE (Isaaks and Srivastava 1989).

2.1.2 Kriging Techniques

The random function $Z(u)$ is expressed into the residual component $R(u)$ and trend component $m(u)$. Depending on the assumption of the trend there are 3 different variations of kriging.

$$Z(u) = R(u) + m(u) \quad (4)$$

2.1.3 Simple Kriging

In this case the trend $m(u)$ of the random function is known and constant across the whole domain A .

$$m(u) = m, \quad \text{constant and known} \quad \forall u \in A \quad (5)$$

2.1.4 Ordinary Kriging

In this case the trend $m(u)$ of the random function is unknown and constant in the local neighborhood $W(u)$.

$$m(u') = m, \quad \text{constant and unknown} \quad \forall u' \in W(u) \quad (6)$$

2.1.5 Universal Kriging or Kriging with Trend

In this case the trend $m(u)$ of the random function is unknown and not constant. The trend varies inside the local neighborhood according to a linear combination of functions $f_k(u)$. a_k is unknown and constant in the local neighborhood $W(u)$.

$$m(u) = \sum_{k=0}^k a_k(u') f_k(u'), \quad \forall u' \in W(u) \quad (7)$$

Among all the kriging methods, ordinary kriging is the most extensively used (Wackernagel 2003). For this reason, in the present study, ordinary kriging was the method chosen to carry out the geostatistical estimates and will be the one described deeply.

2.1.6 Ordinary Kriging Theoretical Framework

For this specific case of kriging method, the local variations of the mean within the domain are accounted by constraining the premise of a constant expectation to the local neighborhood; therefore, as an outcome, the mean will be constant only inside of the local neighborhood $W(u)$ (Goovaerts 1997a; Rossi and Deutsch 2014).

$$m(u) = m(u_\alpha) = m \quad (8)$$

The equation (1) can be written in the following form:

$$Z^*(u) = \sum_{\alpha=1}^{n(u)} \lambda_\alpha(u) Z(u_\alpha) + \left[1 - \sum_{\alpha=1}^{n(u)} \lambda_\alpha(u) \right] \cdot m(u) \quad (9)$$

In order to filter out the unknown mean, the sum of the kriging weights is limited to sum 1.0

$$\sum_{\alpha=1}^{n(u)} \lambda_\alpha^{OK}(u) = 1 \quad (10)$$

Replacing (10) in (9) the Ordinary Kriging estimator is expressed as follows

$$Z_{OK}^*(u) = \sum_{\alpha=1}^{n(u)} \lambda_\alpha^{OK}(u) Z(u_\alpha) \quad (11)$$

Because of equation (10), the requirement of unbiasedness is achieved

$$E\{Z_{OK}^*(u) - Z(u)\} = \sum_{\alpha=1}^{n(u)} \lambda_\alpha^{OK}(u) m(u) - m(u) \quad (12)$$

$$= m(u) - m(u) = 0$$

Knowing that the estimation variance is

$$\sigma_E^2 = E[(Z^*(u) - Z(u))^2] \quad (13)$$

Aiming to satisfy the requirement of minimum variance error the following function must be minimized. To accomplish this, the Lagrange parameter is added to the developed of equation (13):

$$\begin{aligned} \sigma_E^2 = & C_R(u_\alpha - u) + \sum_{\alpha=1}^{n(u)} \sum_{\beta=1}^{n(u)} \lambda_\alpha^{OK} \lambda_\beta^{OK} C_R(u_\alpha - u_\beta) - 2 \sum_{\alpha=1}^{n(u)} \lambda_\alpha^{OK} C_R(u_\alpha - u) \\ & + 2\mu_{OK} \left(\sum_{\alpha=1}^{n(u)} \lambda_\alpha^{OK}(u) - 1 \right) \end{aligned} \quad (14)$$

Where:

- μ_{OK} : Lagrange parameter
- $C_R(h)$: Covariance of the residual function

By reducing the estimation variance to the minimum using the restriction on the weights, the following ordinary kriging system is obtained:

$$\begin{pmatrix} C(u_1 - u_1) & \dots & \dots & C(u_1 - u_n) & 1 \\ \vdots & & & \vdots & \vdots \\ \vdots & & & \vdots & \vdots \\ C(u_n - u_1) & \dots & \dots & C(u_n - u_n) & 1 \\ 1 & & & 1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1^{OK} \\ \vdots \\ \lambda_n^{OK} \\ \mu_{OK} \end{pmatrix} = \begin{pmatrix} C(u_1 - u) \\ \vdots \\ \vdots \\ C(u_n - u) \\ 1 \end{pmatrix} \quad (15)$$

The solve of the previous matrix give the next solution:

$$\begin{aligned} \sum_{\beta=1}^{n(u)} \lambda_{\beta}^{OK}(u) C_R(u_{\alpha} - u_{\beta}) + \mu_{OK}(u) &= C_R(u_{\alpha} - u), \quad \alpha = 1, \dots, n(u) \\ \sum_{\beta=1}^{n(u)} \lambda_{\beta}^{OK}(u) &= 1 \end{aligned} \quad (16)$$

Considering that in practice of ordinary kriging the residual covariance is equal to the global covariance function deduced from all data available, the next expression is achieved:

$$\begin{aligned} \sum_{\beta=1}^{n(u)} \lambda_{\beta}^{OK}(u) C(u_{\alpha} - u_{\beta}) + \mu_{OK}(u) &= C(u_{\alpha} - u), \quad \alpha = 1, \dots, n(u) \\ \sum_{\beta=1}^{n(u)} \lambda_{\beta}^{OK}(u) &= 1 \end{aligned} \quad (17)$$

Replacing equation (17) in (14) the ordinary kriging estimation variance is reduced to the following expression:

$$\sigma_{OK}^2 = C(0) - \sum_{\alpha=1}^{n(u)} \lambda_{\alpha}^{OK}(u) C(u_{\alpha} - u) - \mu_{OK}(u) \quad (18)$$

2.2 Geostatistical Simulation

There are many differences between estimation and simulation methods. While the estimation generates a unique set of estimated values and produces the smoothing effect because of the overstatement of the spatial continuity, the simulation, being a stochastic approach, creates a number of different realizations that keep the statistical and spatial variability of the data samples (Khosrowshahi and Shaw 2001). The simulation reproduces the spatial structure of a data set as a complete group, instead of giving optimum local estimates of the attributes (Ersoy and Yünsel 2006). The simulated realizations that, at a sample location, have a value that matches with the experimental values are called conditional (Goovaerts 1997a).

2.2.1 Conditional Simulation

The Conditional simulation is a Monte Carlo method that generates a number of equally probable scenarios of the grade estimation of the deposit, each of which reflexes the distribution and spatial variability of the data samples (Glacken and Snowden 2001). As mentioned before, a simulation is named conditional if not only the dispersion characteristics

are reproduced (mean and covariance) but also when the simulated values honors and match with the sampled points (Ersoy and Yünel 2006; Chiles and Delfiner 1999).

Considering the random function $Z(x)$ known at N sample points $x_i, i = 1, 2, \dots, N$ and the non-conditional simulation $Z_S(x)$ which, is independent of $Z(x)$ but with the same covariance. The conditioning indicates that the simulated values also match the sample points. This conditioned function will be represented as $Z_{CS}(x)$.

The random function $Z(x)$ can be expressed as follows at the point x at the data $Z(x_i)$:

$$Z(x) = Z_K^*(x) + \{Z(x) - Z_K^*(x)\} \quad (19)$$

Where:

- $Z(x)$: True value
- $Z_K^*(x)$: Kriging estimator
- $Z(x) - Z_K^*(x)$: Kriging error

Because the $Z(x)$ is not known, the kriging error is unknown as well. However, when the same principle is applied to simulation function $Z_S(x)$ in the sample point x_i , where the simulated value is known as well as the kriging estimation value, the following relation will be obtained:

$$Z_S(x) = Z_{SK}^*(x) + \{Z_S(x) - Z_{SK}^*(x)\} \quad (20)$$

Where:

- $Z_S(x)$: Simulated value
- $Z_{SK}^*(x)$: Kriging estimator of simulation $Z_S(x)$
- $Z_S(x) - Z_{SK}^*(x)$: Simulation of kriging error.

In equation (20) the kriging error is known; therefore, it is possible to substitute this value and the kriging estimator from equation (19) to define the conditioned function $Z_{CS}(x)$.

$$Z_{CS}(x) = Z_K^*(x) + \{Z_S(x) - Z_{SK}^*(x)\} \quad (21)$$

Because of the unbiasedness condition of kriging the random function $Z_S(x)$ has the same expectation as $Z(x)$.

$$E[Z_K^*(x)] = E[Z(x)] \quad (22)$$

$$E[Z_{SK}^*(x)] = E[Z_S(x)] \quad (23)$$

Because equations (22) and (23)

$$E[Z_{CS}(x)] = E[Z(x)] \quad (24)$$

Therefore, at location x_i

$$Z_{CS}(x_i) = Z(x_i), \quad i = 1, \dots, n \quad (25)$$

Where:

- $Z(x_i)$: Experimental value of the random function $Z(x)$ at location x_i

2.2.2 Sequential Simulation

Is based in the decomposition of the multivariate probability function of random function $Z(x)$ with a probability density function (pdf) $f(x_1, \dots, x_N; z_1, \dots, z_N)$ in univariate conditionals distribution functions (cdf). This is possible using the Bayes theorem which indicates that the multivariate, with N-dimensional distribution, can be decomposed into a product of N sequentially following univariate conditional distributions. The result of every generation is used to condition the next univariate distribution (Goovaerts 1997a; Gómez-Hernández and Srivastava 2021; Smith and Dimitrakopoulos 1999).

The following expression shows the decomposition of the multivariate conditional probability density function (cpdf) in N univariate cpdfs.

$$f(x_1, \dots, x_N; z_1, \dots, z_N | \Lambda_0) = f(x_1; z_1 | \Lambda_0) \cdot f(x_2; z_2 | \Lambda_1) \dots f(x_N; z_N | \Lambda_{N-1}) \quad (26)$$

Where:

- $\Lambda_0 = \{d_n\} = \{d(x_\alpha), \alpha = 1, \dots, n\}$, original data or conditional data
- $\Lambda_i = \Lambda_{i-1} \cup \{Z(x_i)\}$

The basic steps to execute conditional sequential simulation are the follows (Gotway and Rutherford 1994):

- Define a random path across all grid nodes to establish the sequence in which the conditional univariate distribution will be created.
- Draw a simulated value for the first node, given the N conditioning data.
- Add this simulated value to the conditioning data set.
- Draw a simulated value for the following node, given the N conditioning data and previously simulated values.
- Repeat until all nodes are simulated.

2.2.3 Normal Score Transformation

Given the fact that most of the earth sciences data do not present symmetric gaussian histograms (Deutsch and Journel 1998), to apply correctly the SGSIM method, it is necessary to transform the data into a gaussian distribution.

The normal score transformation aims to transform the RF $Z(x)$ into a new RF $Y(x)$ with a standard normal histogram. The transform is achieved by matching the cumulative distribution function of both Random functions. The process is well explained by Goovaerts (1997).

Considering $\phi()$ as the normal score transform function, the following expression is obtained:

$$Y(x) = \phi(Z(x)) \quad (27)$$

The transform $Y(x) = \phi(Z(x))$, indicates the cumulative probabilities corresponding to $Z(x)$ and $Y(x)$ in p quantiles, reflecting that the distribution has the same value.

$$G(y_p) = F(z_p) = p, \quad \forall p \in [0,1] \quad (28)$$

Having $F(z)$ and $G(y)$ as the cumulative distribution functions of the original RF $Z(x)$ and the standard normal RF $Y(x)$ respectively.

$$F(z) = Prob\{Z(x) \leq z\}$$

$$G(y) = Prob\{Y(x) \leq y\}$$

The following expression represent the transform from the RF $Z(x)$ to RF $Y(x)$

$$Y(x) = G^{-1}[F(Z(x))] \quad (29)$$

$$Z(x) = F^{-1}[G(Y(x))] \quad (30)$$

Where:

- F : Cumulative distribution function of $Z(x)$
- G : Standard normal cumulative distribution function of $Y(x)$

2.2.4 Sequential Gaussian Simulation (SGSIM)

In general, SGSIM performs kriging at every location, from which a simulated value is obtained. In order to ensure that the spatial correlation is kept between simulated points, the previously simulated values are used to condition successive simulated values. The simulation is performed sequentially this means that, at every node, the sample values, originally transformed to normal score, within a neighborhood as well as the previously simulated nodes in the neighborhood, are used to process the kriging estimate values and variance. Every node visited follows a random path in a random order created in each realization. (Ortiz 2020)

Following the same terminology which has been used to describe the above methods, the RF $Z(x_i)$, $x_i \in R, i = 1, \dots, N$, indexed on a discrete grid D_N , and having a conditioning data $d_n = \{z(u_\alpha), \alpha = 1, \dots, n\}$. Following the Conditional simulation and sequential simulation definitions and the kriging technique previously explained, the conditional moments of first and second order as well as the estimator are calculated (Dimitrakopoulos and Luo 2004).

Conditional Expectation:

$$E\{Z(x_i)|\Lambda_{i-1}\} = m_i + C_{i\Lambda_{i-1}} C_{\Lambda_{i-1}\Lambda_{i-1}}^{-1} (Z_{\Lambda_{i-1}} - m_{\Lambda_{i-1}}) \quad (31)$$

Conditional Variance:

$$Var\{Z(x_i)|\Lambda_{i-1}\} = C_{ii} + C_{i\Lambda_{i-1}} C_{\Lambda_{i-1}\Lambda_{i-1}}^{-1} C_{\Lambda_{i-1}i} \quad (32)$$

The realizations $Z(x_i)$ are generated from the following operation

$$Z(x_i|\Lambda_{i-1}) \approx E\{Z(x_i)|\Lambda_{i-1}\} + \sqrt{Var\{Z(x_i)|\Lambda_{i-1}\}} \cdot w \quad (33)$$

Where:

- m_i : Prior expectation of $Z(x_i)$
- C_{ii} : Prior variance of $Z(x_i)$
- $m_{\Lambda_{i-1}}$: Vector of prior means of the conditioned data
- $C_{\Lambda_{i-1}\Lambda_{i-1}}^{-1}$: Inverse matrix of prior covariance between data
- $Z_{\Lambda_{i-1}}$: Conditioning data set Λ_{i-1}
- $C_{i\Lambda_{i-1}}$: Vector of covariance between $Z(x_i)$ and the conditioning data set Λ_{i-1}

- w : Random number $\sim N(0,1)$ independent distributed.

Generally, the SGSIM algorithm involves the following steps:

- The histogram of original data samples must be obtained
- Because the original data from the deposit is not gaussian, the accumulated data is required to be transformed into a gaussian distribution (normalized).
- The normalized experimental variogram is processed. This variogram should have the conditions of mean, variance and sill equal to 0, 1 and 1 respectively. Subsequently, the experimental variogram model is obtained.
- The SGSIM is developed considering the number of realizations to generate.
- Following a random path every node is visited
- Using equation (33) every node obtains a simulated value assigned
- This simulated value is added to the conditioning data set
- The simulation continues until every node is simulated
- The realizations obtained are back transformed to their original units
- It is reviewed that the Exploratory Data Analysis, histogram and variogram are similar to the original data set.

It should be noted that so far these simulations have been developed on a point scale.

2.3 Optimization

The general definition of optimization refers to maximizing or minimizing an objective function considering some specific restriction in the development of the process (Belegundu and Chandrupatla 2019). Applying this concept to engineering the goal will be either to minimize the effort required as well as penalties or to maximize the desired benefit (Rao 2009).

The different kinds of optimization algorithms are related to certain types of problems; however, these could be divided, in general, into two categories: deterministic and stochastic. The main difference between them is that deterministic methods have a fixed path with a given starting point to find the best solution, whereas stochastic methods include a certain level of randomness in the process of finding an acceptable solution (Yang 2010). At the same time, the stochastic methods are divided into two groups: the heuristic and the metaheuristic techniques.

2.3.1 Metaheuristics

The definition of metaheuristics refers to, the generation of iterative processes through a set of specific strategies and models in order to improve the search space and find efficiently near-optimal solutions (Osman and Laporte 1996).

In contrast with the heuristic approach, which is a problem-dependent solution strategy that usually gets stuck in a local optima solution, the metaheuristics, being a problem-independent method, consider approaches to overcome the local optimality trap and can be applied straightforwardly to different problems. (Gendreau and Potvin 2010; Moraga 2016)

Most metaheuristic algorithms are inspired by biological evolution, physical science, nervous systems and statistical mechanics (Osman and Laporte 1996). These algorithms can be classified into 2 groups: population-based and trajectory-based. The main difference between them is that in the first one the algorithm has a set of strings (candidate of solution) to develop the optimization, while the trajectory-based only considers one solution that is iterated through the search space in a fragmented approach (Yang 2010).

Figure 2 shows the summary of the general classification of algorithms previously explained.

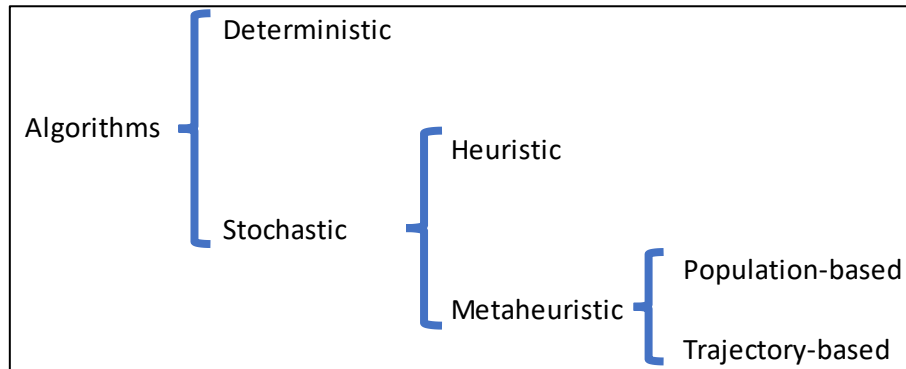


Figure 2: General Algorithms Classification (Yang 2010)

In the present work, because of its simplicity and small time consuming, the stochastic optimization method, Genetic Algorithms (GA), was the chosen method to develop the optimization of the mining scheduling.

2.3.2 Genetic Algorithm (GA)

Published by John Holland in 1975, it is based on Charles Darwin’s theory of natural selection, in which reproduction, crossover and mutation are considered as the basic elements of natural genetics (Rao 2009).

Generation of Initial Population Size

The algorithm works simulating an environment of evolution. First of all, is needed to generate a fixed set of admissible solutions (population size) which will be the individuals of the population, these are represented as chromosomes composed by random bitstrings. The traditional approach to denote chromosomes is the binary string (Figure 3); however, it can be also represented as a n-ary string (Figure 4) (Kochenderfer and Wheeler 2019).

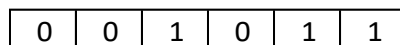


Figure 3: Binary String Chromosome - Example

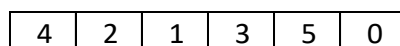


Figure 4: n-ary String Chromosome - Example

Score and Selection of Parents

Following the strategy of “survival of the fittest”, these individuals are subdued to the fitness conditions of the corresponding solutions which are manifested through the objective function that needs to be optimized to obtain a fitness value. Depending on their fitness values

achieved, the weaker individuals are eliminated by the stronger ones in the selection phase of parents (Belegundu and Chandrupatla 2019).

The method to select the parents is the “Tournament selection method” in which the parents are chosen considering the fittest among a random subset of the population, generating a list of parental pairs. The number of parental pairs is the same as the number of initial individuals (Kochenderfer and Wheeler 2019).

Crossover

Between the 2 parents, the crossover is made, exchanging some parts of the chromosomes of them, to generate two offspring. The conventional approach, which works very well with binary chromosomes, indicates to choose and cut a random location of the chromosomes of the parents and exchange that section between them. However, when the individuals are not binary and cannot have repetitions, this technique will generate offspring that will be discarded. The PMX method, developed by Goldberg and Lingle in 1985, allows to exchange one section of the chromosomes without producing repetitions (Üçoluk 2002; Potvin 1996).

Crossover Method: Partially - Mapped Crossover (PMX)

Having chosen 2 parents for the crossover, the PMX method indicates firstly to randomly select one location for the cut points of both parents. Then it is required to select the first bitstring of the second parent and replace it in the same location of the bitstring of the first parent. Due to is highly probable that it will produce a repetition in the first parent, the bitstring replaced is moved to the position of the likely repeated bitstring (Üçoluk 2002; Potvin 1996). This procedure is repeated for both parents until the cut point is reached. Figure 5 and Figure 6 illustrate how the method works.

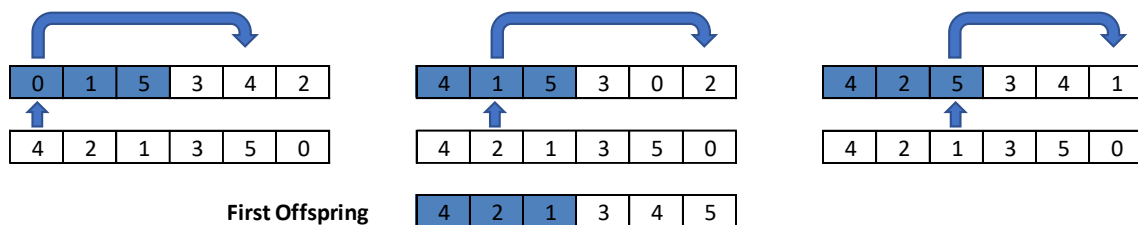


Figure 5: First offspring - Example

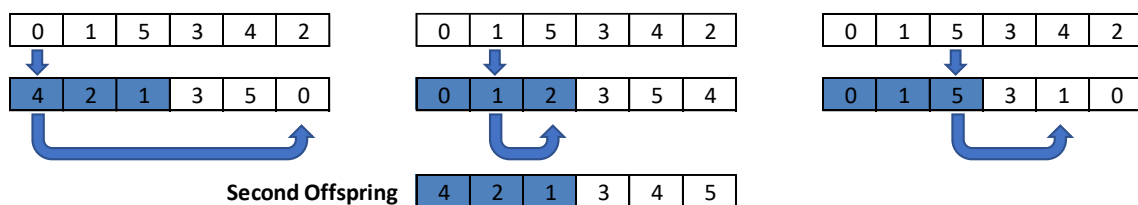


Figure 6: Second offspring - Example

Mutation

Considering that the initial random population does not include all the possibilities or characteristics that may occur, it is necessary that some randomly selected offspring develop a mutation, with which it will be possible to evaluate the fitness of the new traits that it may have, even when it was not generated initially (Kochenderfer and Wheeler 2019).

Mutation Method: Swap

The conventional binary approach considers randomly changing one value of the bitstrings. This technique is not suitable because it will generate repetitions. The swap technique considers in select 2 random bitstrings and exchanges their positions. This technique has the closest strategy to the original approach. (Potvin 1996).

As a summary, the following steps were followed to develop the GA for the present work.

- Define the objective function with the restrictions or fitness functions.
- Generate the initial population size.
- The tournament selection starts to choose the parents, selecting the best fitness values from a subset of the population.
- Once the parents are selected, the crossover is developed using the PMX method. It will allow to generate a new generation of new chromosomes (offspring).
- Some offspring are subdued to the mutation process, which will allow them to explore and obtain better solutions.

2.4 Pillar Dimension

In order to estimate the pillar size, it is required to develop a strength-based design considering the pillar stress (σ_{pa}) and pillar strength (σ_p). The Safety Factor (SF) defines the relation of the pillar load capacity (pillar strength) divided by the pillar load (pillar stress), as can be seen in equation (34). An acceptable SF depends on the tolerable risk of failure. In the case of pillars in main development headings or panels during advance mining a SF of, at least, 2 is conventional and acceptable (Bullock 2011). However, since the calculations made are based on assumed values and formulas from literature, a safety factor of 3.5 as minimum will be considered to overcome deviations from the theoretical formulas.

$$SF = \frac{\sigma_p}{\sigma_{pa}} \quad (34)$$

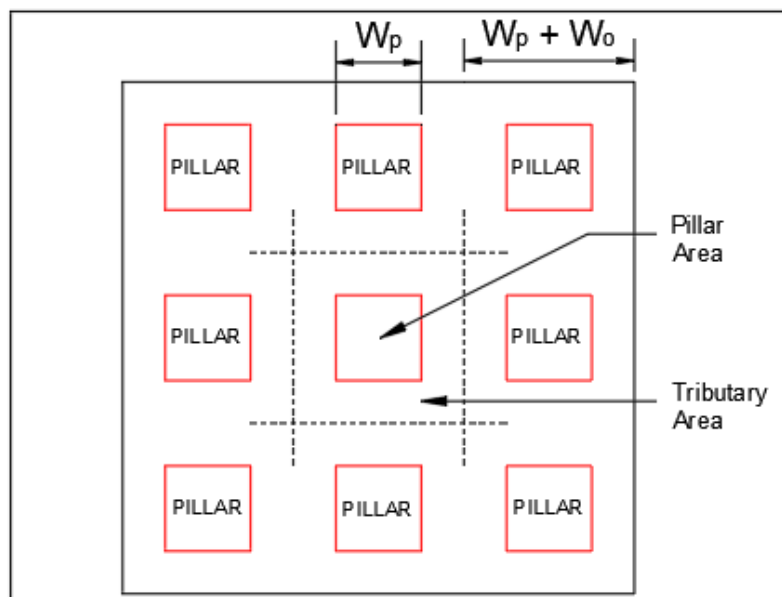


Figure 7: Plan view of room and pillar mine design (Zipf 2001)

2.4.1 Pillar Stress (σ_{pa})

It is required calculate first the in-situ vertical stress (σ_s), which is the product of the rock average unit weight (γ) and the depth of the mining horizon (z) (Farmer 1992).

$$\sigma_s = \gamma * z \quad (35)$$

The average pillar stress (σ_{pa}) is estimated using the tributary area method. This method assumes that the pillars have the same dimensions and the mined area is extensive. Despite of the fact that this is a very simple technique, for the purpose of this study is acceptable for a straightforward estimation. Considering Figure 7, the average pillar stress for a square room and pillar mining is defined in equation (36) (Zipf 2001).

$$\sigma_{pa} = \sigma_s * \left(\frac{W_p + W_o}{W_p} \right)^2 \quad (36)$$

Where:

- W_p : Pillar width
- W_o : Opening width

2.4.2 Pillar Strength (σ_p)

There are different traditional empirical formulas to estimate the strength of a pillar. Due to the fact of using a synthetic data set, some established values from the data set need to be assumed. For this reason, the Obert-Duvall method (equation (37)) was chosen since it was developed for all rock types and at the same time only requires the Unconfined Compressive Strength (UCS) value of the intact rock (Kendorski 2007).

$$\sigma_p = \sigma_c * \left(0.78 + 0.22 * \frac{W}{H} \right) \quad (37)$$

Where:

- σ_c : UCS of the defined sample
- W : Pillar width and length
- H : Pillar height

2.4.3 Area Extraction Ratio (r)

With the pillar width already defined, the area extraction ratio can be calculated, which is determined by the mined area divided by the total orebody area (Brady and Brown). Following the elements of the mining system of Figure 7, the expression is reduced to equation (38).

$$r = 1 - \left(\frac{W_p}{W_p + W_o} \right)^2 \quad (38)$$

The Area Extraction Ratio is an important parameter, in order to quantify the amount of ore that can be extracted per block.

2.5 Cut-off Grade

The basic definition of cut-off grade refers to a criterion broadly used in mining to classify waste and ore of a deposit (Lane 1964). There are different approaches to calculate the cut-off grade; however, for the purpose of the present study case, it was selected the most simplistic method which is the “Breakeven Cut-off grade”.

2.5.1 Breakeven Cut-off Grade

The Breakeven is a straightforward method to calculate the cut-off grade. Due to the fact that it only considers financial parameters it is considered as a one-dimensional process. It only includes direct cost, price and ore recovery; however, the geology and capacity constraints are not considered (Hall 2014; Githiria and Musingwini 2018).

Equation (39) represents the general equation to calculate the Breakeven cut-off grade. In this formula, the cut-off is obtained by equalizing the utility for mining ore and waste (Hall 2014; Rendu 2014).

$$x_c = \frac{(M_o + P_o + O_o) - (M_w + P_w + O_w)}{r * (V - R)} \quad (39)$$

Where:

- r : Ore recovery, %
- V : Price, USD/tn-metal
- R : Refining cost, USD/tn-metal
- M_o : Mining cost, USD/tn-ore
- P_o : Processing cost, USD/tn-ore
- O_o : Overhead cost, USD/tn-ore
- M_w : Mining cost, USD/tn-waste
- P_w : Processing cost, USD/tn-waste
- O_w : Overhead cost, USD/tn-waste

Depending on the costs to consider in the analysis, the breakeven cut-off can be implemented for different assessments of the mining operation. It is important to mention that there are no standard names to define the cut-off in the different aspects of the mining operation. The cut-offs that are being considered are the following:

2.5.2 External or Mine Cut-off Grade without Waste Stripping

This cut-off includes the costs of mining and processing only the ore (Rendu 2014). In this scenario, the waste material is not extracted so it does not have an assigned cost as shown in equation (40).

$$x_c = \frac{M_o + P_o + O_o}{r * (V - R)} \quad (40)$$

2.5.3 Marginal Cut-off Grade

In this case, the cut-off formula, in equation (41), only considers the variable cost (Hall 2014). The material above the marginal cut-off and below the Mine cut-off grade is the material used for blending to keep feeding the plant with a regular grade and tonnage. Any grade below the marginal cut-off grade is considered uneconomical.

$$x_c = \frac{Mv_o + Pv_o}{r * (V - R)} \quad (41)$$

Where:

- r : Ore recovery, %
- V : Price, USD/tn-metal
- R : Refining cost, USD/tn-metal
- Mv_o : Variable mining cost, USD/tn-ore
- Pv_o : Variable processing cost, USD/tn-ore

3 A New Method for Short-Term Extraction Sequencing under Uncertainty

In this chapter, the detailed methodology will be used to develop a short-term production program that meets all the established operational and economic requirements. This will be achieved following the flowchart shown in Figure 8.

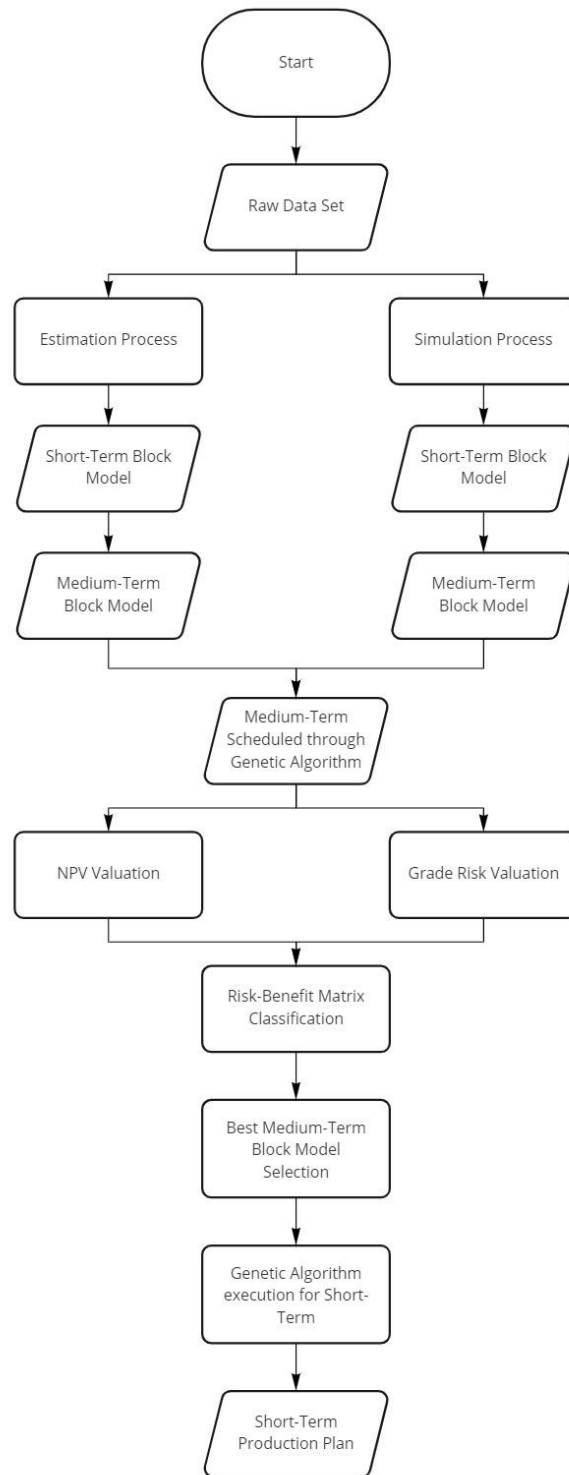


Figure 8: Flow Chart - General Methodology

Specifically, the methodologies developed to be able to generate the estimated and simulated block models, as well as the optimization of the mining sequencing, will be presented. In addition, the logic behind the risk and benefit classification will be developed, as well as the weighted evaluation for each sequencing by developed block model.

For the exploratory data analysis, variogram and block models generation, the Stanford Geostatistical Modelling Software (SGeMS) has been used. The SGeMS is an open-source computer software that provides enough basic tools needed to reach the purpose of the present research.

The optimization will be developed using the new approach developed of the Genetic Algorithm technique conditioned to the “room and pillar” mining method. Aiming to achieve this, it will be necessary to develop a code that allows creating stochastic sequencing of the block models, for this purpose the Python programming language will be used

The valuation and classification will be developed using the Gaussian function (bell curve) properties, as well as a risk matrix.

3.1 Data set Description

For the purpose of this study case, the information of the drill holes is considered as a synthetic data set, which will represent a tabular deposit of copper with a dip of 30° in sedimentary competent rock. For these conditions, the mining method chosen was “Cut and Fill - Post Pillar (Room and Pillar with backfilling)”.

3.1.1 Setup of Deposit Conditions

The following assumed conditions are required to be able to define the pillar width and the Extraction Area Ration as well.

Table 1: Setup of Deposit Conditions

Parameter	Value
Rock type	III A
Deposit Depth (m)	200
Density Overlaying rock (Kg/m3)	2300
UCS pillar, MPa	110

Following the scheme of Figure 7 and replacing the information from Table 1 in equations (35), (36) and (37), the value obtained of the pillar width is 4.0 meters providing a Safety Factor of 3.9. Once the pillar width is defined is possible to calculate the Area Extraction Ration using equation (38), this value being 0.84. This means that only 84% of a block is extracted.

3.1.2 Setup of Operative Conditions

Considering that only 84% of the content of the blocks can be extracted, for the purpose of this work, the production program must be a multiple of 84. For this reason, the daily production ratio will be 1680 tons.

3.1.3 Setup of Economic Conditions

In order to define a representative mining operation, it will be necessary to establish the assumed costs as regular and standard as possible. Therefore, for practical purposes, it will be used standard average cost from literature.

Using the standard costs from Table 2 (Hustrulid and Bullock 2001), it is possible to extrapolate the values to the capacity desired of 1680 tn/day, as is expressed in Table 3.

Table 2: Baseline – Mine Operating Costs for “Room and Pillar”

Parameter	Cost/unit	Cost/unit
Production, tn/d	1,200	8,000
Stopes, USD/tn	5.62	4.47
Drifts, USD/tn	3.05	1.26
Crosscuts, USD/tn	2.09	0.92
Ventilation Raises, USD/tn	0.08	0.20
Main Haulage, USD/tn	2.57	1.22
Services, USD/tn	2.04	1.25
Ventilation, USD/tn	0.12	0.07
Maintenance, USD/tn	2.54	0.99
Administration, USD/tn	4.39	1.81
Total Operating Costs, USD/tn	22.78	12.31

Source: Underground Mining Methods (Hustrulid and Bullock 2001)

Table 3: Mine Operating Costs for “Room and Pillar”

Parameter	Cost/unit
Production, tn/d	1,680
Stopes, USD/tn	5.34
Drifts, USD/tn	2.61
Crosscuts, USD/tn	1.80
Ventilation Raises, USD/tn	0.11
Main Haulage, USD/tn	2.24
Services, USD/tn	1.85
Ventilation, USD/tn	0.11
Maintenance, USD/tn	2.16
Total Operating Costs, USD/tn	16.20

Aiming to estimate the processing cost, it will be necessary to use a relation between mining operating cost and milling operating costs for 1680 tn. For the purpose of the study case, the following empirical formulas, shown in equations (42) and (43) can be used (Camm 1991).

The processing cost using the Flotation Mill model with one product is expressed as follows:

$$P_o = 121 * (x^{-0.335}) \quad (42)$$

Where:

- P_o : Processing cost, USD/tn-ore
- x : Capacity of mill in short tons per day

Mining cost for “Room and Pillar” method:

$$M_o = 35.5 * (x^{-0.171}) \quad (43)$$

Where:

- M_o : Mining cost, USD/tn-ore
- x : Capacity of mine in short tons per day

Developing the calculation and changing the units to MT, the following relation is obtained:

$$\frac{P_o}{M_o} = 0.957\% \quad (44)$$

Having defined the information from Table 3, using equation (44) and following equation (40) the Mine Cut-off Grade is estimated, as can be seen in Table 4.

Table 4: Economic Parameters - Mine cut-off grade

Parameter	Value
Mining cost, USD/tn-ore	16.20
Processing cost, USD/tn-ore	16.08
Overhead cost, USD/tn-ore	5.25
Ore recovery, %	90.2%
Price, USD/tn-metal	7000*
Refining cost, USD/tn-metal	80*
Cut-off	0.60%

(*) Historical average information.

Considering equation (41) the Marginal Cut-off Grade is calculated and expressed in Table 5.

Table 5: Economic Parameters – Marginal cut-off grade

Parameter	Value
Mining cost, USD/tn-ore	16.20
Processing cost, USD/tn-ore	16.08
Overhead cost, USD/tn-ore	-
Ore recovery, %	90.2%
Price, USD/tn-metal	7000.00
Refining cost, USD/tn-metal	80.00
Cut-off	0.52%

(*) Historical average information.

3.2 Exploratory Data Analysis (EDA)

Exploratory data analysis (EDA) is the first step to understand the dataset's statistical behaviour. The analysis considers a univariate distribution, which is the copper grade. Based on this will be developed statistical evaluations, to gain insight on trends and patterns of the ore body. Is useful to have a visual representation of the information, so the basic tool for the EDA is the histogram. Along with the histogram, the statistical indicators of central parameters of distribution (mean, median and mode), general parameters of a distribution (lower quartile, upper quartile, minimum and maximum) and parameters of distribution for dispersion (variance) should be obtained. It is important to mention that the exploratory data analysis is performed considering the whole deposit and not only of the chosen level.

3.3 Analysis of Spatial Variability

This analysis is critical to measure the local variability of the data, which is vital to define the spatial continuity of an attribute, in the present study case, copper grade. The semi-variogram is the tool used to describe this spatial continuity of a characteristic. It determines the average dissimilarity between data points separated by a distance h (Goovaerts 1997b). It is expressed by the following formula:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{\alpha=1}^{N(h)} [z(u_{\alpha}) - z(u_{\alpha} + h)]^2 \quad (45)$$

Having the experimental omnidirectional variogram processed and with it the nugget effect, sill and range values obtained, is possible to generate the variogram model. Depending on the variability of the data, the variogram model could be spherical, exponential or Gaussian.

3.4 Operation Level Setting

Because of the size of the deposit, the amount of data and computer capacity constraints, the procedures will be developed only in one level of the mineral deposit to be developed a 2D model. This is completely acceptable since the objective of the work is to generate a short-term mining plan that considers the geological uncertainty in it. Therefore, it will not be

necessary to make a block model of the entire deposit, but rather a single exploitation level will be established. The 2D model will reduce the requirement of the computer's processing capacity in the simulations. At this level, the EDA should be developed as well.

3.5 Ordinary Kriging – Point Scale Estimation

Using the variogram model, the process of estimation starts. The estimation method selected was Ordinary Kriging, because as mentioned before, OK is the most widely used estimation method. The development of the OK estimation follows the flow chart expressed in Figure 9.

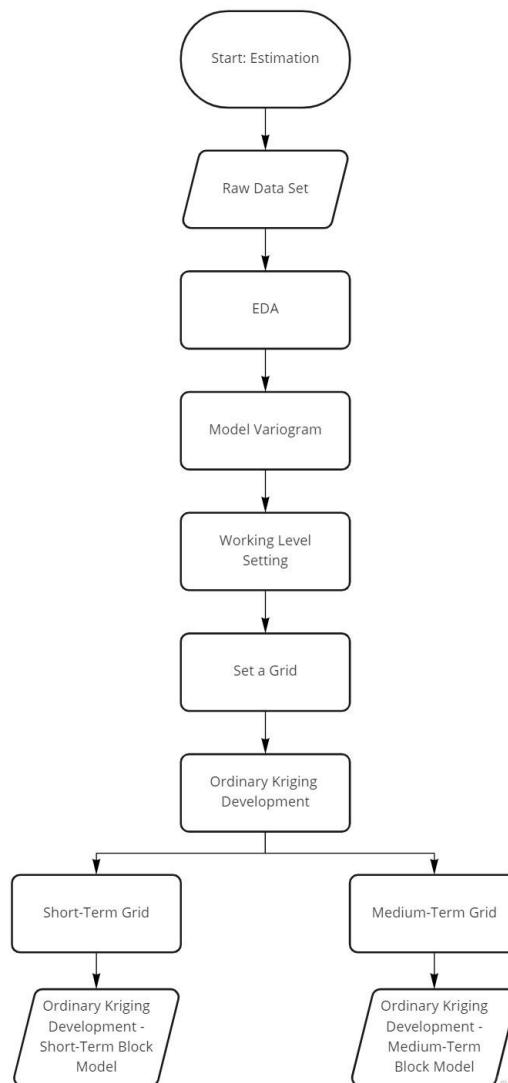


Figure 9: Flow Chart – Ordinary Kriging Estimation

Once the OK is executed the analysis of the first results is carried out by comparing the estimated statistical indicators with the original ones. It is important to point out the presence of the smoothing effect and the effects in the variance.

The previous ordinary kriging estimation was developed on a point scale. To create a block model, these points values must be averaged to block size to subsequently be used as base data to calculate the ordinary kriging in SMU size.

3.6 Ordinary Kriging – Short-Term Block Model Estimation

After the moving average point grid is developed a new grid for the block model must be created. The dimensions of the grid nodes should be of the desired size of the block dimension, this means that each block must have the dimensions of the Selective Mining Unit (SMU). The ordinary kriging is developed using this time the point grid moving average as hard data and with it, the estimated block model is generated.

3.7 Simulation – Sequential Gaussian Simulation (SGSIM)

The development of the SGSIM follows the sequence detailed in the flow chart in Figure 10 and is detailed in the following sub-chapters as well.

3.7.1 Normal Transformation

Knowing that Conditional Simulations assume a gaussian distribution of data, the input data set must be transformed to normal distribution. The software SGeMS uses the tool “trans” to transform the raw data set to a normal distribution with a mean value of 0 and variance of 1.

Having the normalized data set, the Analysis of Spatial Variability is developed as well as in the estimation method, obtaining the omnidirectional variogram which will allow to evaluate the variability of the property selected.

3.7.2 Sequential Gaussian Simulation – Point Scale Simulation

Sequential Gaussian simulation was the method chosen to develop the simulation, because of its simplicity and, due to the fact that when performing a 2D simulation, the data processing will not require a high capacity from the computer and will not take long to execute.

The simulation models in a point scale are obtained after the simulations processed are back transformed.

3.7.3 Sequential Gaussian Simulation – Short-Term Block Model

As with the estimation method, the previous simulations were developed on a point scale. Aiming to create block models, these point values must be averaged to block size (SMU). Through the previously explained algorithm “Moving Window – Moving Average”, the new point grids are generated and, as well as, in the estimation case, the Ordinary Kriging is developed using these points as hard data generating the block models. All of the previous block models generated are considered Short-Term block models.

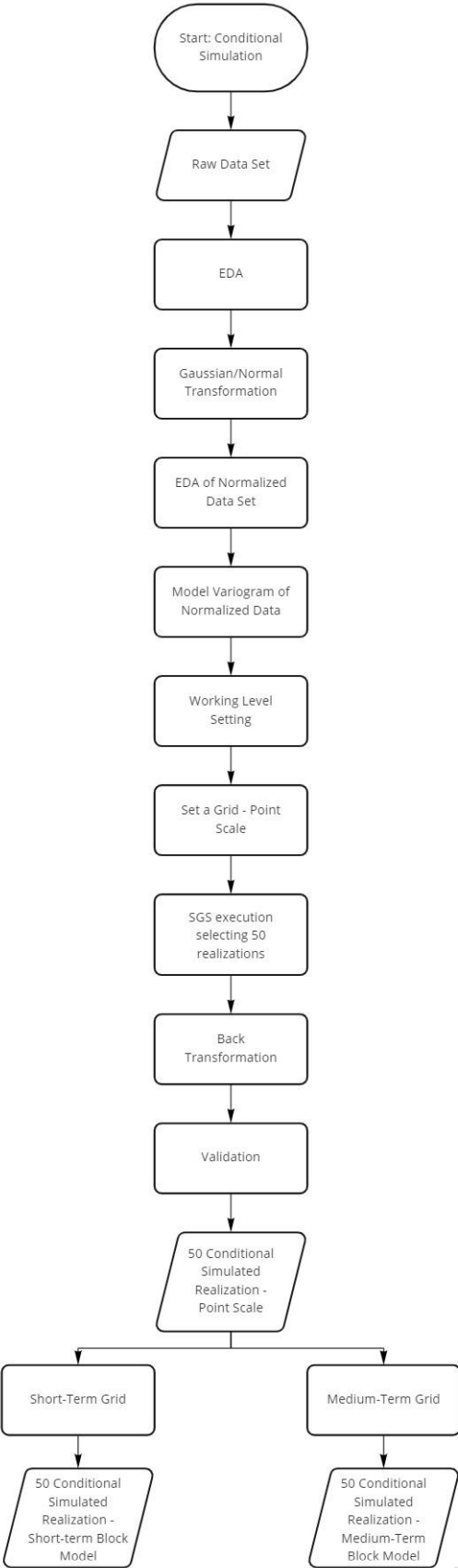


Figure 10: Flow Chart – Sequential Gaussian Simulation

3.8 Medium-Term Block Model

Since the previously generated Short-Term block models contain a large amount of information, it is required to develop a block model with larger blocks, which will produce a lower processing load for the computer hardware when running algorithms in the next stage. This new configuration of the block model will be considered Medium-Term.

In order to develop this new block model, it is required to create a new grid with the same limits as the previous ones, but with a larger block size. The following step is to execute the OK using the point grid moving average as hard data, as developed with the short-term block model. The outcome will be block models with larger blocks but less quantity, generating more manageable data.

Having all the estimated and simulated block models available, as well as the cut-off grade and the pillar dimensions, the analysis of the tonnage grade curve is developed.

3.9 New Optimization Approach - GA Modified for “room and pillar” Method

Utilizing the programming language python, the conditions and requirements to develop the algorithm that will allow obtaining extraction sequences in a short time are defined and implemented, following the flow chart detailed in Figure 11. In this part, the following scripts adapt the Genetic Algorithm method to the "room and pillar" mining method.

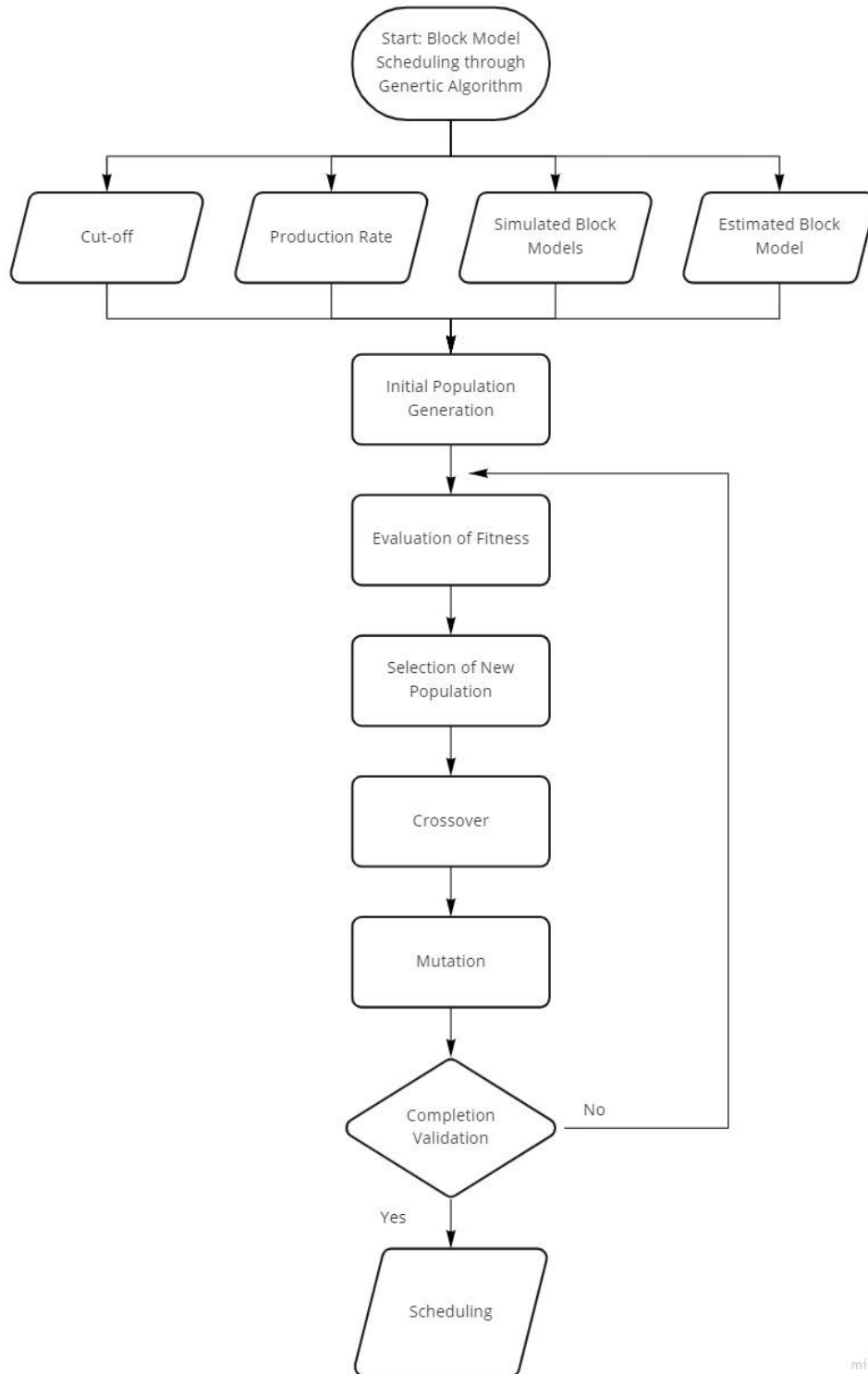


Figure 11: Flow Chart – Scheduling through Genetic Algorithm Modified

3.9.1 Population Generation

The population is considered as a set of random bitstrings with integer values from zero to “number of blocks - 1”. The initial population is created as is shown in Figure 12, having two hyperparameters on it, the “n_bits” and “n_pop”. The “n_bits” defines the number of bits in the array, which refers to the initial random sequence of block extraction. The n_pop defines the size of the population (number of arrays generated), which refers to the set of initial possible mining extraction sequences.

```
# initial population of random bitstring
pop = [np.random.permutation(n_bits).tolist() for _ in range(n_pop)]
```

Figure 12: GA – Initial population

3.9.2 Objective Function

The objective function is a two-stage function that minimizes the total penalty generated from the deviations in meeting the requirements of operation and sequence. The first stage refers to the penalty for not meeting the cut-off grade. The second stage penalizes when the mining extraction does not follow a logical sequence. When the penalties are higher, the higher the fitness score, therefore the lower the feasibility of the solution.

3.9.3 Penalty Grade

Per period, there is a set of blocks that need to be extracted in order to meet the daily requirement of mine production and grade. This stage works with the hyperparameter “cut_off”. Depending on whether the scheduling will be developed for the medium or short-term, these requirements must be adjusted. However, the logic is the same, the average grade of the blocks per period will be calculated and a penalty directly proportional to the square difference with the cut-off grade will be added (Equation (46)). The minimum penalty_grade considered is 10. Figure 13 shows the algorithm behind this explanation.

$$\text{penalty_grade} = (\text{average_grade} - \text{cutoff})^2 * 1000 \quad (46)$$

```
#Objective Function
def objective(pop):

    ### Restrictions deviation from average cut-off ###

    grades = []
    i = 0
    while i < len(pop) :
        current_grade = grades_i[pop[i]]
        grades.append(current_grade)
        i = i+1

    average_grade = []
    j = 0
    while j < len(pop) :
        current_average_grade=float(format(((grades[j] + grades[j+1] + grades[j+2] + grades[j+3] + grades[j+4])/n_blocks),".6f"))
        j = j + n_blocks

    penalty_list = []
    for element in average_grade :
        element = ((element - cut_off)**2)*1000
        element_1 = float(format(element, ".6f"))
        penalty_list.append(element_1)

    penalty_grade = sum(penalty_list)+10
```

Figure 13: GA – Penalty_grade

3.9.4 Penalty Order

This penalty is related to the location of the blocks in the sequence of extraction. This algorithm calls the hyperparameter “n_block”, which is used to identify how many blocks should be mined per period. The mining extraction sequence considered in this case is from south to north, so the blocks must be extracted considering the following conditions:

- A block can be mined if the previous block (on vertical axis) was already mined.
- A block can be mined if the immediate block next to it was already mined.

There is a penalty each time that blocks do not accomplish these conditions. However, when all the conditions are met, the penalty is 1.0.

Equation (47) shows the penalty in case the first block is not in the first row of the block model

$$Penalty = Penalty + 50000 \quad (47)$$

Equation (48) shows the penalty in case the mined block does not meet the two previously indicated conditions. In this case, the further north the block is, the greater the penalty.

$$Penalty = 100 * \left(\frac{\text{Bitstring block value}}{\text{Number of rows in the BM}} \right) \quad (48)$$

```

### Restrictions mining sequence from South to North ###

penalty_list_1 = []
penalty_1=1
for block in range(0,len(pop)):
    if pop[block] <= len(set(x_i)) - 1:
        penalty_1 = 1
    elif (pop[block] >= len(set(x_i)) and block == 0) :
        penalty_1 = penalty_1 + 50000
    else:
        for a in range (0, block):
            if ((pop[block]-pop[a]!=len(set(x_i)))or((abs(pop[block]-pop[a])==1)and(pop[block]%len(set(x_i))))and a<block :
                penalty_1 = 1
                break
            elif pop[block] - pop[a] != len(set(x_i)) and a < block :
                penalty_1 = 100*(math.floor(pop[block]/len(set(x_i))))
        penalty_list_1.append(penalty_1)
penalty_order = float(format(sum(penalty_list_1), ".6f"))

```

Figure 14: GA – Penalty_order

Equation (49) shows the total penalty, which is just the sum of the “penalty_order” and “penalty_grade”. Figure 15 shows the algorithm behind this expression.

$$Total_penalty = penalty_order + penalty_grade \quad (49)$$

```

## Total Penalty ##

penalty = float(format((penalty_order + penalty_grade), ".6f"))

return penalty

```

Figure 15: GA - Total_penalty

3.9.5 Tournament Selection (parents)

The parents are chosen using the tournament method already explained. The parent selected will be the ones with a lower score (minimizing). Figure 16 explains the algorithm that allows to obtain the parents with a better score.

```

#tournament selection
def selection(pop, scores, k=3):
    #first random selection
    selection_ix = randint(len(pop))
    for ix in randint(0, len(pop), k-1):
        #check if better
        if scores[ix] < scores[selection_ix]:
            selection_ix = ix
    return pop[selection_ix]

```

Figure 16: GA – Tournament selection (parents)

3.9.6 Crossover – PMX Method

Figure 17 shows the crossover function with the PMX method among the parents. This method avoids repetition of values and is activated when a random probability is less than the hyperparameter “r_cross”

```

#Crossover - PMX method
def crossover(p1, p2, r_cross):
    c1, c2 = p1.copy(), p2.copy()
    c3 = p1.copy()
    if rand() < r_cross:
        elements_cross = randint(0, len(p1)-1)
        for i in range(0, elements_cross):
            c1[c1.index(c2[i])] = c1[i]
            c1[i] = c2[i]
        for j in range(0, elements_cross):
            c2[c2.index(c3[j])] = c2[j]
            c2[j] = c3[j]
    return [c1, c2]

```

Figure 17: GA – Crossover

3.9.7 Mutation -Swap Method

Figure 18 shows how works the mutation function, selecting 2 random bitstrings and exchanging their positions. The hyperparameter “r_mut” activates the mutation, while a random probability is less than “r_mut”.

```

#Mutation - Swap method
def mutation(bitstring, r_mut):
    for i in range(len(bitstring)):
        if rand() < r_mut:
            i1 = randint(0, len(bitstring)-1)
            i2 = randint(0, len(bitstring)-1)
            while i2 == i1:
                i2 = randint(0, len(bitstring)-1)

            a = bitstring[i1]
            b = bitstring[i2]
            bitstring[i1] = b
            bitstring[i2] = a

```

Figure 18: GA – Mutation

3.9.8 Genetic Algorithm Function

Joining all the functions previously explained, the Genetic Algorithm function is developed. This function depends on the hyperparameters: n_bits, n_pop, n_iter, r_cross, r_mut, cutoff and n_blocks. It is important to mention that the hyperparameter n_iter (number of iterations) creates a loop that generates a number of generations of offspring equal to the n_iter value.

```

# Genetic Algorithm
def genetic_algorithm(objective, n_bits, n_iter, n_pop, r_cross, r_mut):
    time_elapsed = time()

    # initial population of random bitstring
    pop = [np.random.permutation(n_bits).tolist() for _ in range(n_pop)]

    #keep track on the best solution
    best = 0
    best_eval = objective(pop[0])

    #enumerate generation
    for gen in range(n_iter):
        #evaluate all candidates in the population
        scores = [objective(c) for c in pop]
        print(gen)

        #check for new best solution
        for i in range(n_pop):
            if scores[i] < best_eval:
                best, best_eval = pop[i], scores[i]
                print(">Xd, new best f(%s) = %.3f" % (gen, pop[i], scores[i]))

        #select parents
        selected = [selection(pop, scores) for _ in range(n_pop)]

        #create the next generation
        children = list()
        for i in range(0, n_pop, 2):
            # pair selected parents
            p1, p2 = selected[i], selected[i+1]

            # Crossover and mutation
            for c in crossover(p1, p2, r_cross):

                # Mutation
                mutation(c, r_mut)
                # store for next generation
                children.append(c)
        # replace population
        pop = children

    print("time elapsed:",time()-time_elapsed)
    print(gen)
    return [best, best_eval]

```

Figure 19: GA - Function

3.10 Mine Planning Scheduling

Owing to the fact that developing mining scheduling is time-consuming, especially if it has to be developed for each generated simulation, the new optimization approach developed in the present work called is an alternative to overcome this difficulty.

In order to use the Genetic Algorithm modified for room and pillar efficiently, the procedure will be divided into 2 phases.

- The first phase is the medium-term planning in which the level selected is scheduled in a mesh with block dimension of medium-term size to schedule.
- Once the optimum extraction path is selected, the algorithm is executed again for every medium-term block individually, to develop the short-term planning with blocks of SMU size.

The outcome will be the mining extraction scheduling of every block model and with them, it would be possible to calculate and compare the Net present value and classify the models.

3.11 Net Present Value (NPV)

Aiming to calculate the NPV from every medium-term block model, it is necessary to have defined the operative and economic factors. The operative factors allow to define the metallic content that can be extracted using a mining method defined, while the economic factors will allow to define the payable ore as well as revenues and costs.

3.11.1 Operative Factors

The parameters needed to define the configuration of the blocks for the mining method defined, in this case, "room and pillar" are the following:

- Short-term block dimension, m

- Medium-term block dimension, m
- Medium-term block volume, m^3
- Density, t/m^3
- Pillar dimension, %

Equation (51) shows the procedure to calculate the extractable metal content from each block.

$$Ore(tn) = Block_Vol (m^3) * Density (tn/m^3) \quad (50)$$

$$Metal(tn) = Ore(tn) * (1 - Pillar(\%)) \quad (51)$$

Equations (52) and (53) shows the total amount of ore and metallic content respectively, in tones per period t .

$$Total_Ore^t = \sum_{t=0}^p O^t \quad (52)$$

Where:

- p : number of scheduling periods
- O^t : Ore in tones in period t

$$Total_Metal^t = \sum_{t=0}^p M^t \quad (53)$$

Where:

- p : number of scheduling periods
- M^t : Metallic content in tones in period t

3.11.2 Economic Factors

The parameters needed in this section will define the value of the block model and with it their economic viability. These are the following:

- Mining cost ($Mining_cost$), USD/t
- Ore processing cost ($Processing_cost$), USD/t
- Ore recovery %
- Smelting and refining cost (SR_Cost), USD/t-metal
- Discount rate (i), % per year
- Discount rate (i_p), % per Period (medium-term)
- Cold Prices, Cu USD/t-metal
- Cash-cost ($Cash_cost$), USD/tn ore
- Cut-off

Using equations (53) and (51) the payable metal in a period t is calculated as follows:

$$P_Metal^t = Total_Metal^t * Ore_recovery(\%) \quad (54)$$

The cash-cost is calculated as the expression (55)

$$Cash_cost = Mining_cost + Processing_cost \quad (55)$$

The revenue and costs in USD per period are expressed in the expressions (56), (57) and (58)

$$Revenue^t = P_Metal^t * Cold_Price(USD/tn\ metal) \quad (56)$$

$$Cost^t = Cash_cost * Total_Ore^t \quad (57)$$

$$SR_Cost^t = SR_cost * P_Metal^t \quad (58)$$

The cash flow per period (equation (59)) is calculated and with it, the NPV of each block model is obtained, following equation (60).

$$Cash_flow^t = Revenue^t - Cost^t - SR_Cost^t \quad (59)$$

$$NPV = \sum_{t=0}^P \frac{Cash_flow^t}{(1+i)^t} \quad (60)$$

3.12 Risk-Benefit Valuation

Once the NPV of the whole set of block models is calculated, it is required to classify and select the best model among them to could develop the short-term planning. Having this objective, it is necessary to develop a risk assessment valuation of the outcomes already obtained.

The valuation aims to choose the best block model among the realizations developed. The best block model means that the simulation selected must have the highest profit with the lowest geological risk. To accomplish this, NPV and Grade Risk valuations will be assessed. It is important to mention that the valuations are measured in percentage.

The valuation of the NPV and the Grade Risk will allow to understand and classify the different block models generated with a valuation factor, the Risk-benefit valuation, which is only the product of both previously named valuations calculated.

$$RB_VAL(x) = NPV_VAL(x) * Grade_risk_VAL(x) \quad (61)$$

3.12.1 NPV Valuation

The valuation of the NPV, expressed in equation (63) is in general simple. It considers as higher the NPV, the valuation will be higher. Establishing that the mean of the NPV value of the set of simulations, represented in equation (62), has a 50% of valuation, the proportion of the NPV with respect to the mean will define the valuation. The application of equation (63) allows to classify which values have the highest NPV.

$$NPV(sim\bar{x}) = \frac{1}{N} \sum_{i=1}^N NPV(simx_i) \quad (62)$$

Where:

- $NPV(simx_i)$: NPV per simulation i

$$NPV_VAL(x) = \frac{NPV(simx) * 0.5}{NPV(sim\bar{x})} * 100 \tag{63}$$

The values obtained per iteration receive a classification according to Table 6. The mean value, that has 50%, is considered as “Moderate” risk.

Table 6: NPV Risk Classification

Minimum		Moderate		High		Maximum	
0%	40%	40%	60%	60%	80%	80%	100%

3.12.2 Grade Risk Valuation

The geological uncertainty is measured having as a base the metal content. For this situation, the valuation considers as closer to the Metal content mean value as higher the valuation. In addition, this first valuation is penalized by a correction factor that is related to the Gaussian function properties.

The correction factor is calculated considering the following circumstances, which are summarized in Table 7:

If a value of the Metallic content is inside 0.5 standard deviations of the mean, the correction is 38.2%, which is the proportion of values that are located in this range.

If a value of the Metal content is between 0.5 and 1.0 standard deviations of the mean, the correction is 30.0%, which is the proportion of values that are located only in this range.

If a value of the Metal content is between 1.0 and 2.0 standard deviations of the mean, the correction is 27.2%, which is the proportion of values that are located only in this range.

If a value of the Metal content is outside 2.0 standard deviations of the mean, the correction is 4.4%, which is the proportion of values that are located outside of this range.

Table 7: Correction Factor for Geological Risk

Type	Distribution	Correction Factor (CF)
Very High	$> \mu - 3\sigma$	4.40%
	$< \mu + 3\sigma$	
High	$> \mu - 2\sigma$	27.20%
	$< \mu + 2\sigma$	
Intermediate	$> \mu - \sigma$	30.00%
	$< \mu + \sigma$	
Low	$> \mu - 0.5\sigma$	38.20%
	$< \mu + 0.5\sigma$	

Equation (64) shows the mean value of the metallic content, which is the baseline to define grade risk valuation shown in equation (65).

$$MC(sim\bar{x}) = \frac{1}{N} \sum_{i=1}^N MC(simx_i) \tag{64}$$

Where:

- $MC(simx_i)$: Metallic content per simulation i

$$Grade_risk_VAL(x) = \left(1 - \frac{MC(sim\bar{x}) - MC(simx)}{MC(sim\bar{x})} \right) * CF \tag{65}$$

3.13 Risk Matrix

The Block Model with a higher Risk-benefit valuation is the one that has a better balance between NPV potential and grade risk. However, depending on the economical requirements and willingness of the company in assumes risks, it is possible to choose a Block model with a lower RB_VAL . For this reason, the complete set of block models are classified in the risk matrix, shown in Table 8, depending on the valuations previously calculated.

Table 8: Risk Matrix

Grade Risk			NPV							
			Minimum		Moderate		High		Maximum	
Type	Distribution	Correction Factor	0%	40%	40%	60%	60%	80%	80%	100%
Very High	$> \mu - 3\sigma$ $< \mu + 3\sigma$	4.40%								
High	$> \mu - 2\sigma$ $< \mu + 2\sigma$	27.20%								
Intermediate	$> \mu - \sigma$ $< \mu + \sigma$	30.00%								
Low	$> \mu - 0.5\sigma$ $< \mu + 0.5\sigma$	38.20%								

3.14 Short-term Planning Scheduling

Once the Optimum Block model is selected, the following activity is to obtain the short-term scheduling. Aiming to accomplish this, the GA modified for “room and pillar” will be used again in the SMU block size, following the sequence already developed by the previous medium-term sequence. With the complete scheduling is possible to develop the daily and monthly mining plan.

4 Demonstration in a Controlled Environment

In the present chapter, following the methodology previously explained, the study case will be established, considering all the operational and economic assumptions, in order to process the information and be able to develop the block models, scheduling and risk-benefit valuations. The outcome will be the daily production program accomplishing the operational requirements using as input the block model with the highest economic potential and lowest geological risk.

4.1 Exploratory Data Analysis – Cu Deposit

The information shown in the histogram, which is skewed to the right, reveals that the maximum and minimum Cu grade value 1.02% and 0.51 respectively, with a mean grade of 0.66% and it is. The total statistical information can be seen in Table 9.

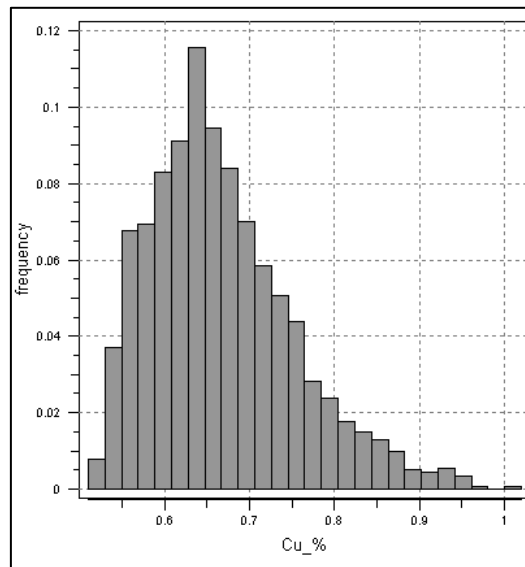


Figure 20: Histogram – Cu deposit

Table 9: Statistical Parameters Cu Deposit

Statistical Parameter	Value
Mean grade (Cu %)	0.665
Variance	0.008
Minimum	0.51
Lower quartile	0.60
Median	0.65
Upper quartile	0.71
Maximum	1.02

Once the information from the EDA is obtained, the variogram of the deposit is developed, as can be shown in Figure 21. The onmi-directional variogram shows a spherical behaviour and its parameters are detailed in Table 10. Following the information from Table 10, the variogram model is constructed as Figure 21 shows as well.

Table 10: Variogram Parameters Raw data

Variogram Parameter	Value
Nugget effect	0.003
Sill contribution	0.004
Type	Spherical
Range (m)	960

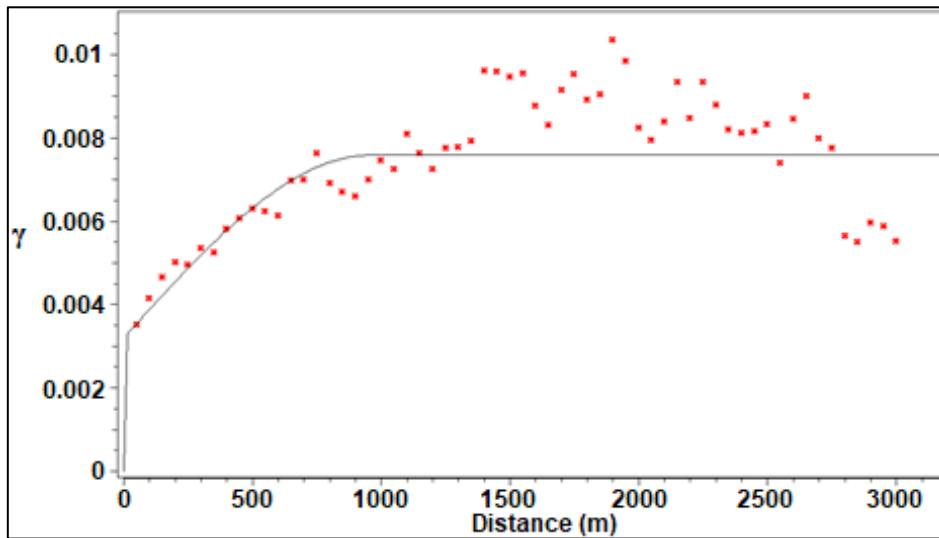


Figure 21: Omni-directional Variogram – Raw data

4.2 Working Level – Lv. 5020

In order to exemplify the processes to be developed as well as to reduce the requirement of the processing capacity of the computer, the simulations will be developed in 2D. To perform this, one single level of the entire deposit will be selected, which, with the focus on being representative, must have a certain level of dispersion in its values. For this reason, the selected level was Lv. 5020 in which the grade of Cu varies between 0.52 and 0.82 as can be seen in Figure 22

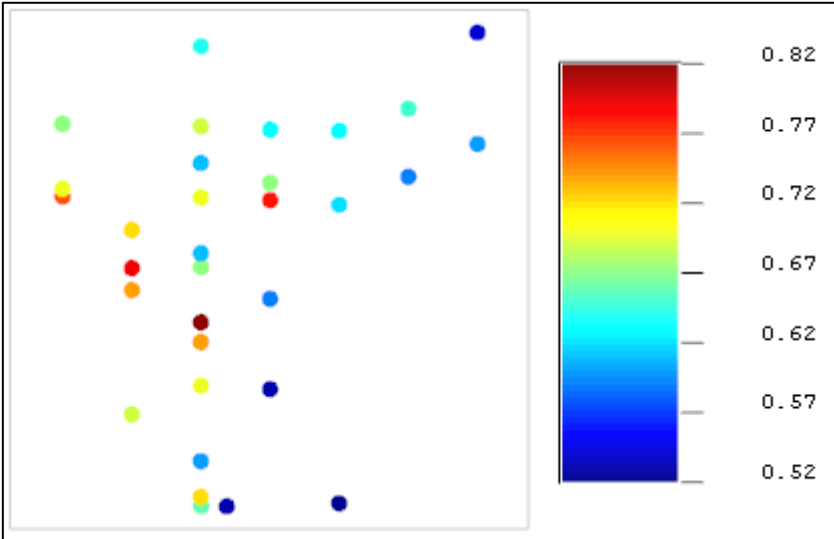


Figure 22: Data set Lv. 5020 in 2D view

The statistical parameters and the histogram shown in Table 11 and Figure 23 reflect the information of the chosen level. These parameters will be essential to compare the estimated and simulated models with the original data set at the working level.

Table 11: Statistical Parameters Lv. 5020

Statistical Parameter	Value
Mean grade (Cu %)	0.657
Variance	0.006
Minimum	0.52
Lower quartile	0.60
Median	0.66
Upper quartile	0.70
Maximum	0.82

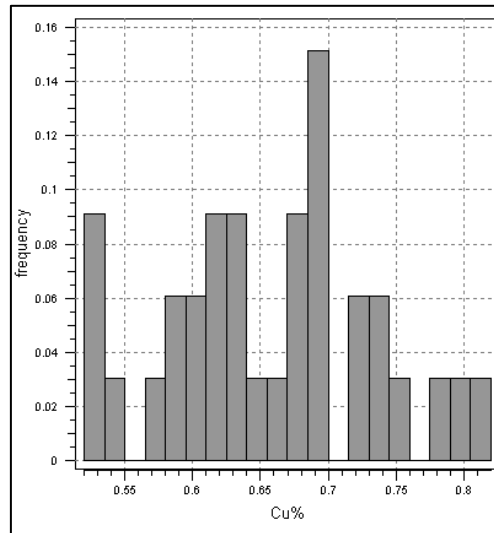


Figure 23: Histogram – Lv. 5020

4.3 Ordinary Kriging Estimation

4.3.1 Point Scale Model

In order to develop the Ordinary Kriging, the information required by the software interface must be completed.

Initially, the grid must be delimited with dimensions that cover all the samples, in this case the grid will have dimensions of 300x300 m² with 1.0x1.0 m² cells. In the kriging interface, the “General and Data” section contains the information of the grid, the hard data, and the search ellipsoid (Figure 24). At the same time, the information used for the “Variogram” section is the one previously modelled (Figure 25). The range considered was 2000 aiming to cover the complete grid section, and the number of points considered to develop the estimation varies between 5 and 20 points. The information added in the variogram section is the one from Table 10.

Figure 24: Interface – Estimation / kriging / General and Data

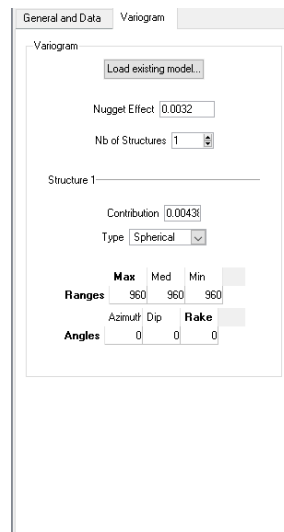


Figure 25: Interface – Estimation / kriging / Variogram

The Ordinary Kriging estimation obtained can be appreciated in Figure 26. The deterministic method reveals that there is a zone with a high-grade concentration in the west zone. However, in the east zone, the grades are low. It should also be stated that in the southeast zone, more drilling needs to be carried out to have a greater certainty of the grade in that area. This is reflected because of the high variance values in this zone as shown in Figure 27.

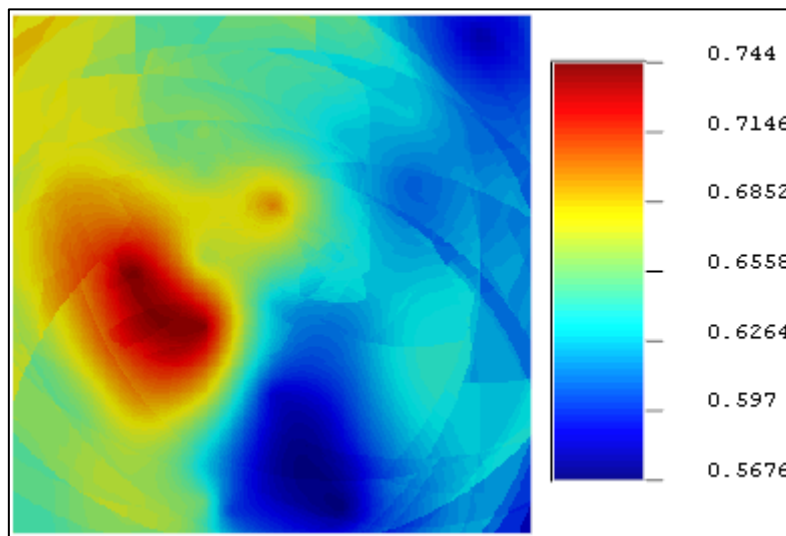


Figure 26: Ordinary Kriging Estimation – Point Scale

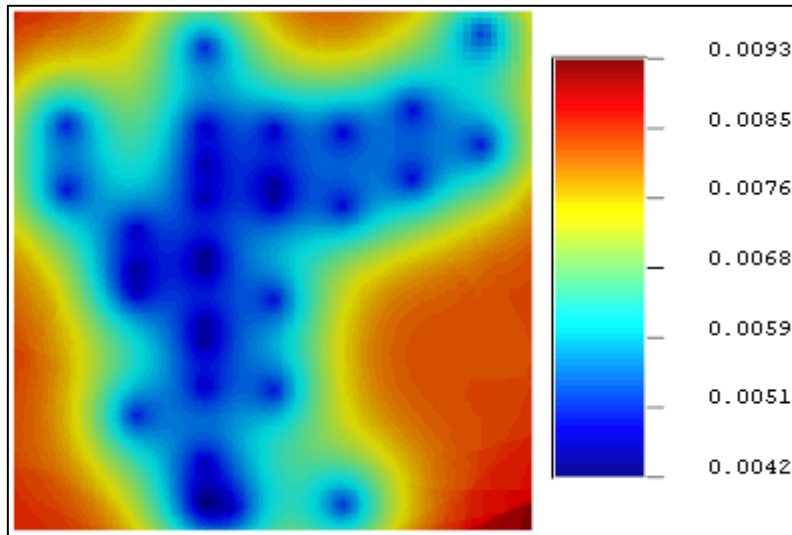


Figure 27: Variance Ordinary Kriging Estimation – Point Scale

4.3.2 Short-Term Block Model

The previous ordinary kriging estimation was developed in a point scale, in order to create a block model these points values must be averaged to a point grid of 1.0x1.0 m² to subsequently be used as a base data to calculate the ordinary kriging in SMU size of 10x10 m².

The function of the SGeMS that allows to execute this is the “Moving Window – Moving Average”. The Moving Average calculates the linear average in the neighborhood, this averaged value is positioned in the window central point. In the interface, shown in Figure 28, the “window geometry” must be defined (Remy et al. 2009).

The screenshot shows a software interface for configuring a 'Moving Window' operation. The interface is organized into several sections:

- Input:** Contains two dropdown menus. The 'Object' dropdown is set to 'OK_10x10' and the 'Property' dropdown is set to 'OK_5-20_2000'.
- Prefix for output:** An empty text input field.
- Filter Type:** A dropdown menu set to 'Moving Average'.
- Window Geometry:** Contains two radio buttons: 'Rectangular' (which is selected) and 'Ellipsoid'.
- Half template size:** A sub-section with three input fields for 'X', 'Y', and 'Z'. The 'X' field contains '10', the 'Y' field contains '10', and the 'Z' field contains '0'.

Figure 28: Interface – Utilities / Moving Window

Figure 29 shows the Ordinary Kriging after the moving average. Since the values were averaged with the next 1.0x1.0 grid nodes, this model seems to be smoother than the model developed in Figure 26.

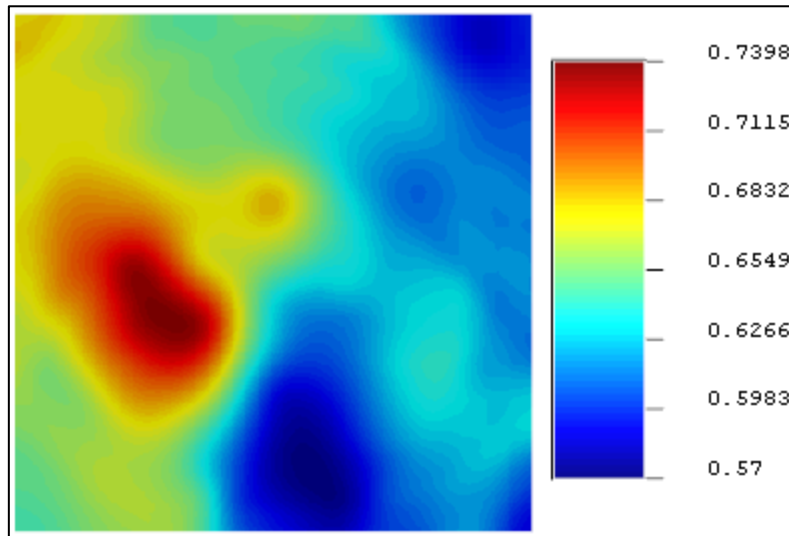


Figure 29: Ordinary Kriging Estimation – Moving Average Point Grid

A new grid is created with grid node dimensions equal to the SMU in this case of 10.0x10.0 m² and the block model is created as in Figure 30. Considering that the size of the grid is 300x300 m², this means that the short-term block model will have 900 blocks.

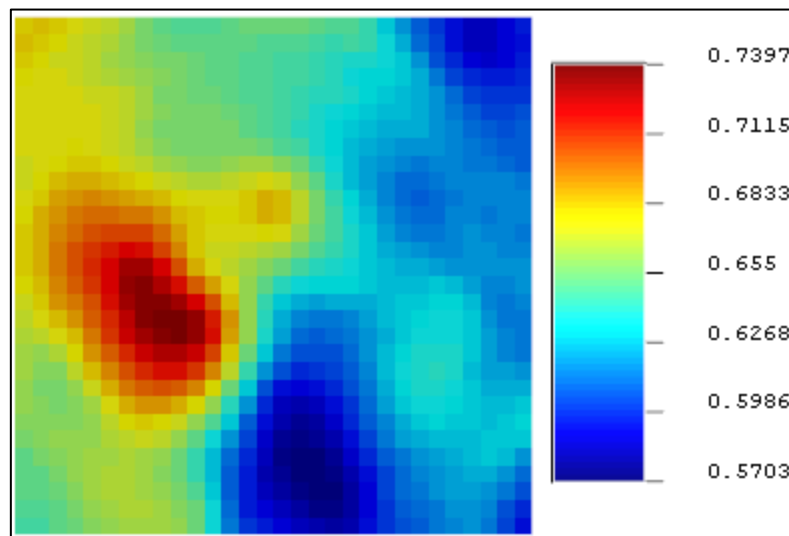


Figure 30: Ordinary Kriging Estimation – Short-Term Block Model

4.3.3 Medium-Term Block Model

It was decided to generate the medium-term block model with the objective of using a smaller number of blocks and thus having a more manageable data size to process. The construction of the medium-term block model implies to create a new grid. Aiming to keep the blocks as regular as possible, the new cell sizes will be 60x60 m². This means that the model will have 25 blocks on it and for every medium-term block, there will be 36 short-term blocks as is shown in Figure 31. This configuration will also be used to develop the medium-term models with Gaussian simulations as well.

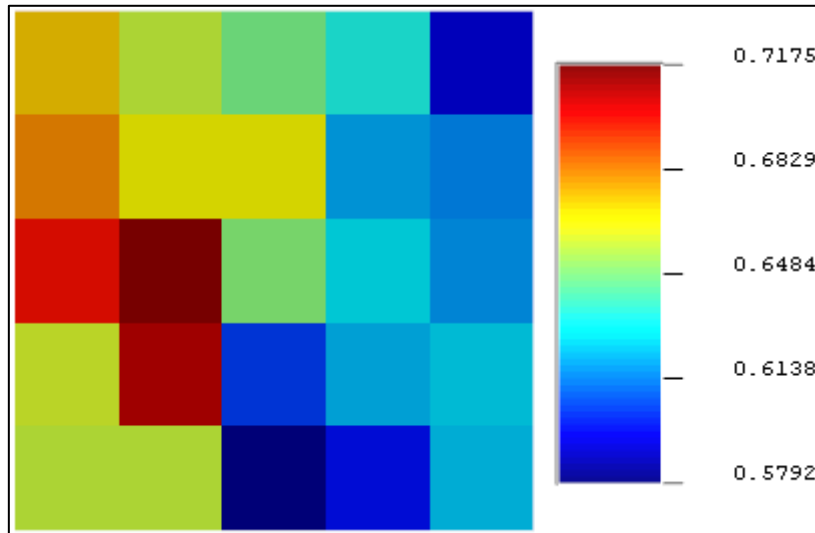


Figure 31: Ordinary Kriging Estimation – Medium-Term Block Model

4.4 Sequential Gaussian Simulation (SGSIM)

To develop the grade uncertainty measurement, the Kriging estimation will be compared with the stochastic simulations using the sequential Gaussian simulation (SGSIM) method. In this case study, 50 realizations were developed, since being a hypothetical case study, this number of simulations will be enough to demonstrate the deviations of the deterministic model.

Following the steps detailed in Chapter 3.7, the raw data set should be normalized. To develop the transformation, the information required by the software interface must be completed. The “Data” section contains the information of the property to normalize, which in this case is the copper grade (Figure 32). Likewise, the “Source” section, in Figure 33, requires the type of information with which it is working and also requires the minimum and maximum values of the property to be evaluated (copper grade), which will be defined when the EDA is developed later in chapter 4.1. The last section “Target” contains the type of transformation to perform, in this case, is a Gaussian type with mean and variance of 0 and 1 respectively (Figure 34).

Figure 32: Interface – Utilities / trans / Data

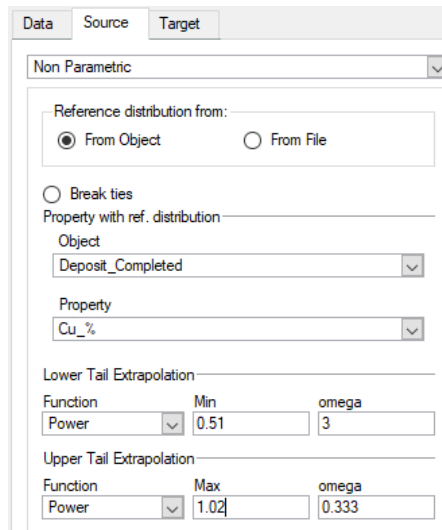


Figure 33: Interface – Utilities / trans / Source

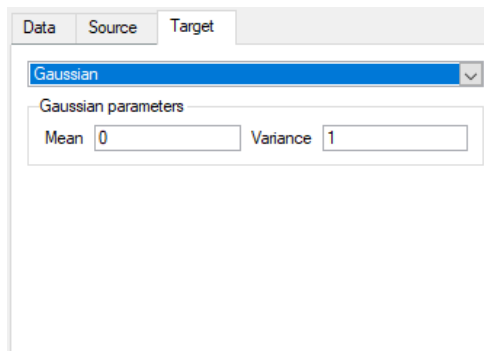


Figure 34: Interface – Utilities / trans / Target

The histogram of the Normal transformation can be seen in Figure 35. In Table 12 can be reflected the characteristics of a gaussian distribution, since the mean is close to zero and the variance almost 1.0. Subsequently, the variogram is developed.

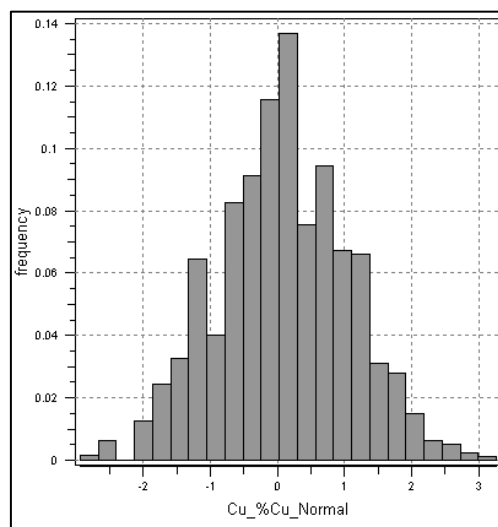


Figure 35: Histogram – Normalized Data Set of Cu deposit

Table 12: Statistical Parameters Normalized Data Set

Statistical Parameter	Value
Mean grade (Cu %)	0.065
Variance	0.947
Minimum	-2.935
Lower quartile	-0.628
Median	0.051
Upper quartile	0.682
Maximum	3.261

The omni-directional variogram shows a spherical behaviour and its parameters can be seen in Table 13. In Figure 36 the black line represents the variogram model that describes the experimental variogram (red crosses) with a nugget effect of 0.35, sill contribution of 0.65 and range of 1530.

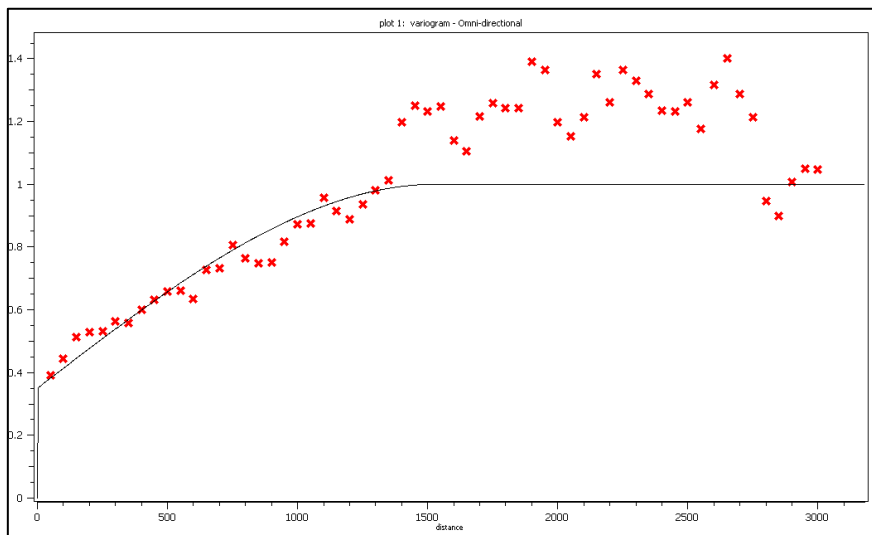


Figure 36: Omni-directional Variogram – Normalized Data

Table 13: Variogram Parameters – Normalized data

Variogram Parameter	Value
Nugget effect	0.350
Sill contribution	0.650
Type	Spherical
Range (m)	1530

4.4.1 Point Scale Model

Having the grid dimension of 300x300 m² with cells sizes of 1.0x1.0 m², the 50 realizations were carried out in the Lv. 5020. The search ellipsoid considers 1580 in order to cover all the grid section.

The sequential Gaussian simulation is developed using the tool “sgsim” from SGeMS. As in the previous method, a grid must be established. In order to make a comparison between simulated and estimated models, a grid of the same dimensions as the one used for Ordinary Kriging should be considered and the information required in the SGSIM interface must be completed as well. The “General” section requires the number of simulations to develop and the type of kriging to consider (Figure 37). The following section is “Data” which needs the normalized data set and the ranges of the search ellipsoid (Figure 38). The last section, shown in Figure 39, is “Variogram” in which the information of the normalized variogram must be included (Remy et al. 2009).

The screenshot shows the 'General' tab of the SGSIM interface. It features three tabs: 'General', 'Data', and 'Variogram'. The 'Simulation Grid' section includes a dropdown for 'Simulation Grid Name' (set to 'SGSIM_10x10_50sim_SK') and a text input for 'Property Name Prefix' (set to 'SGSIM_SK'). Below this, there are three input fields: 'Nb of realizations' (50), 'Seed' (14071789), and 'Kriging Type' (Simple Kriging (SK)).

Figure 37: Interface – Algorithm / sgsim / General

The screenshot shows the 'Data' tab of the SGSIM interface. It features three tabs: 'General', 'Data', and 'Variogram'. The 'Hard Data' section includes dropdowns for 'Object' (Cu_Deposit_Level_5020) and 'Property' (Cu%_Normalized), with a checked checkbox for 'Assign hard data to simulation grid'. The 'Search Ellipsoid' section includes a dropdown for 'Max conditioning data' (12) and a table for 'Ranges' and 'Angles'. The 'Target Histogram' section has an unchecked checkbox for 'Use Target Histogram'.

	Max	Med	Min
Ranges	1580	1580	1580
Angles	0	0	0

Figure 38: Interface – Algorithm / sgsim / Data

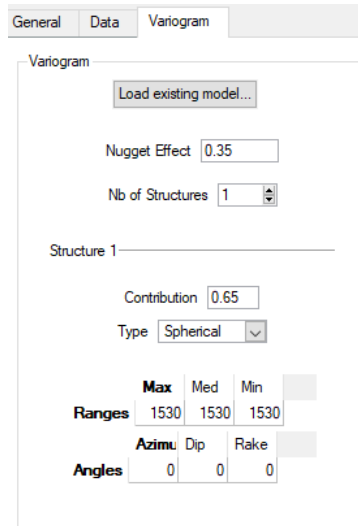


Figure 39: Algorithm / sgsim / Variogram

The outcomes are normalized simulations which subsequently must be back transformed to non-parametric distribution with the same tool “trans” only exchanging the information between Source and Target sections.

Knowing that the simulations must reproduce the original histogram at Lv. 5020 and, at the same time, maintain the variability of the samples, the validation of all 50 realizations can be undertaken by comparing the simulated histograms (Figure 40, Figure 41, Figure 42, Figure 43, Figure 44, Figure 45, Figure 46, Figure 47, Figure 48, Figure 49, Figure 50, Figure 51, Figure 52) with the original data set in the same level 5020 (Figure 23) which must be similar.

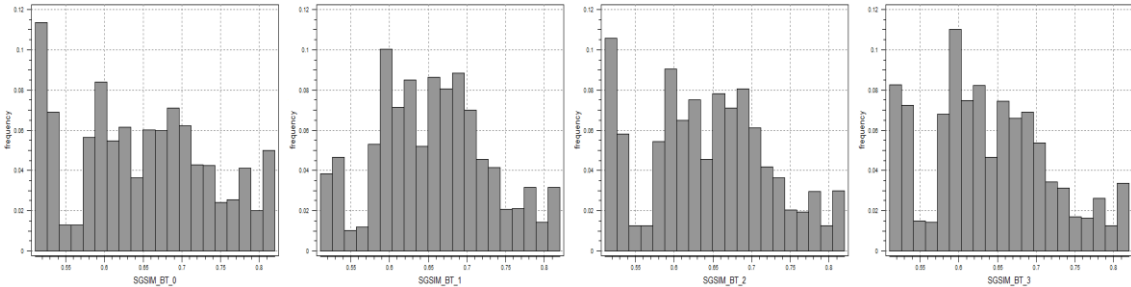


Figure 40: Histograms – Sim_0, Sim_1, Sim_2, Sim_3

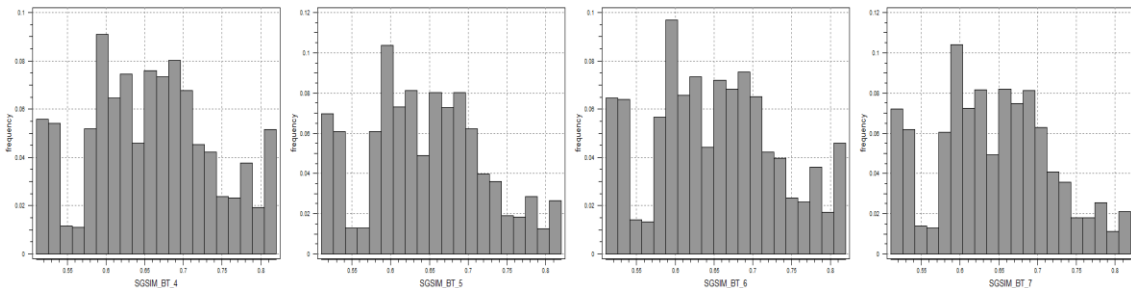


Figure 41: Histograms – Sim_4, Sim_5, Sim_6, Sim_7

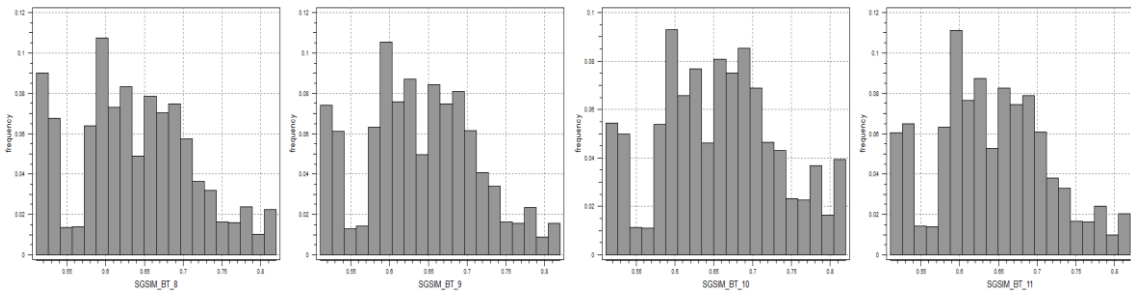


Figure 42: Histograms – Sim_8, Sim_9, Sim_10, Sim_11

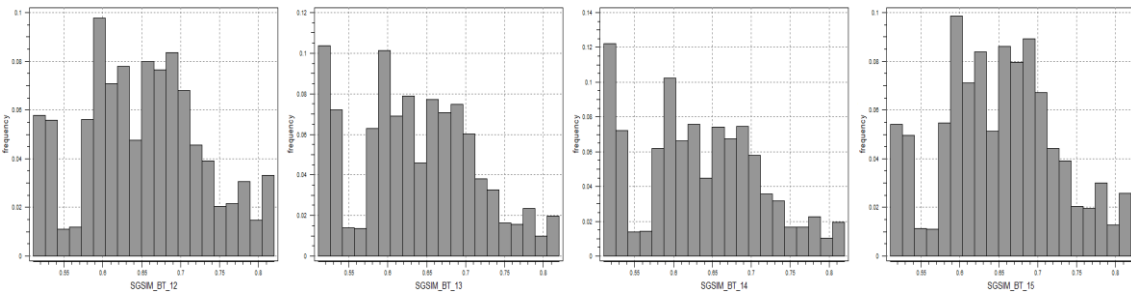


Figure 43: Histograms – Sim_12, Sim_13, Sim_14, Sim_15

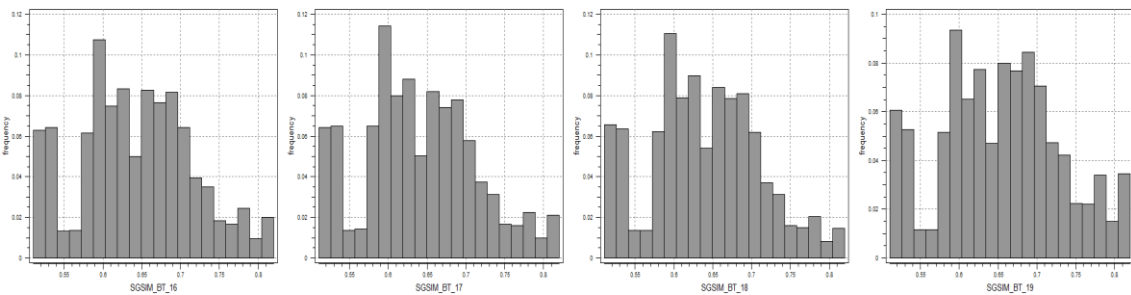


Figure 44: Histograms – Sim_16, Sim_17, Sim_18, Sim_19

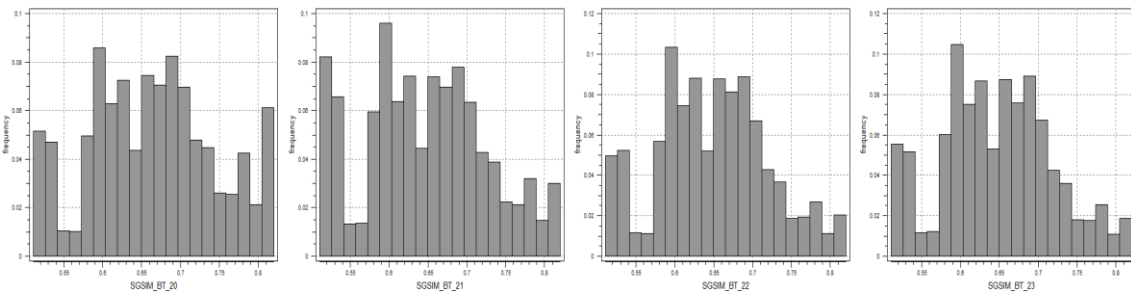


Figure 45: Histograms – Sim_20, Sim_21, Sim_22, Sim_23

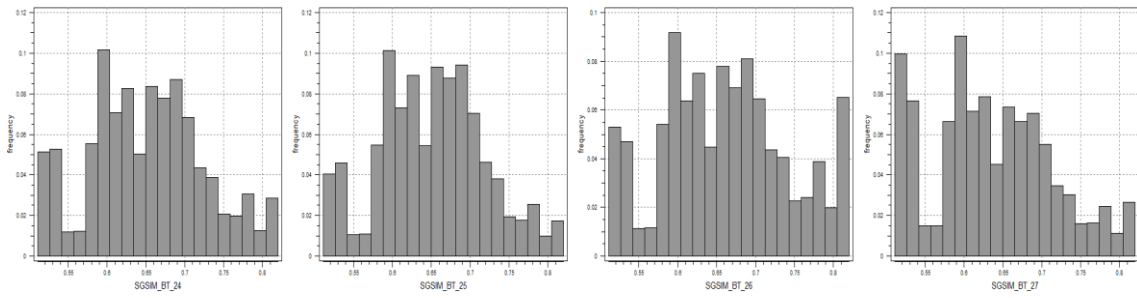


Figure 46: Histograms – Sim_24, Sim_25, Sim_26, Sim_27

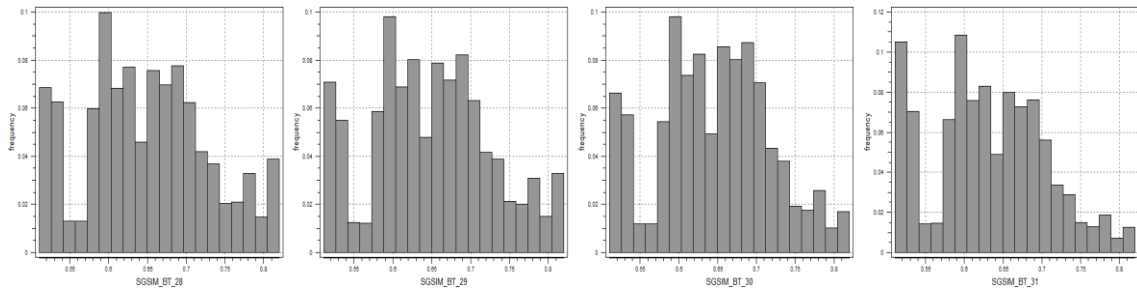


Figure 47: Histograms – Sim_28, Sim_29, Sim_30, Sim_31

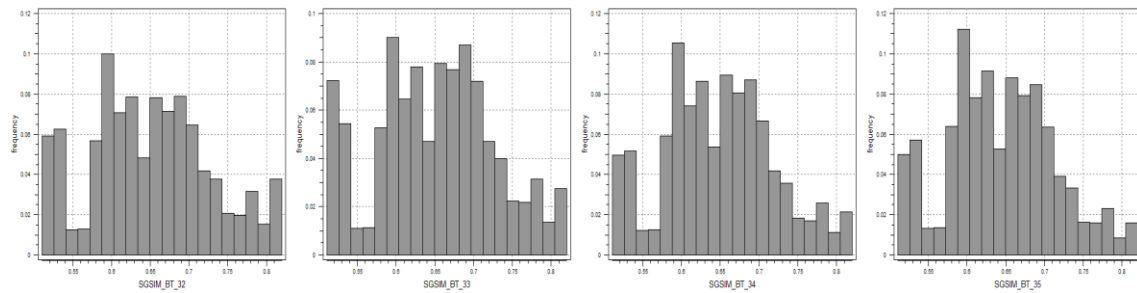


Figure 48: Histograms – Sim_32, Sim_33, Sim_34, Sim_35

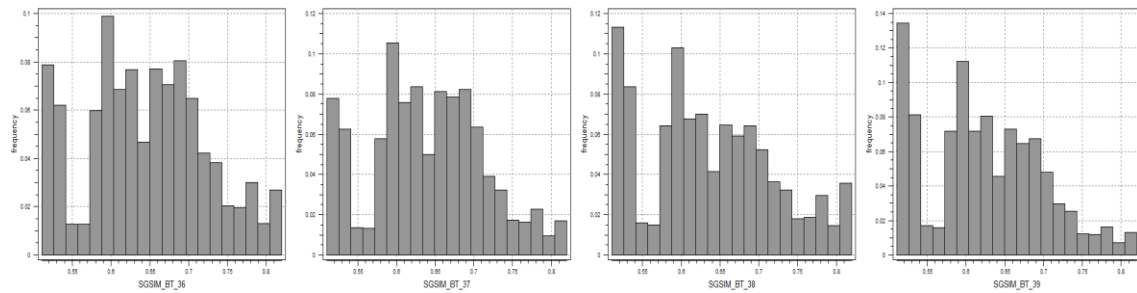


Figure 49: Histograms – Sim_36, Sim_37, Sim_38, Sim_39

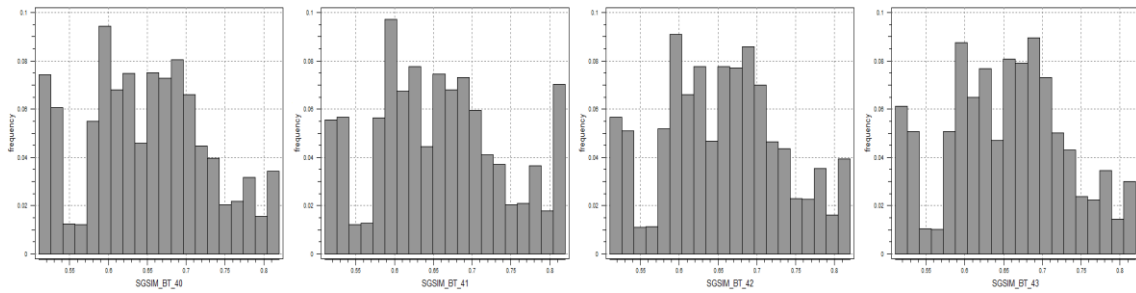


Figure 50: Histograms – Sim_40, Sim_41, Sim_42, Sim_43

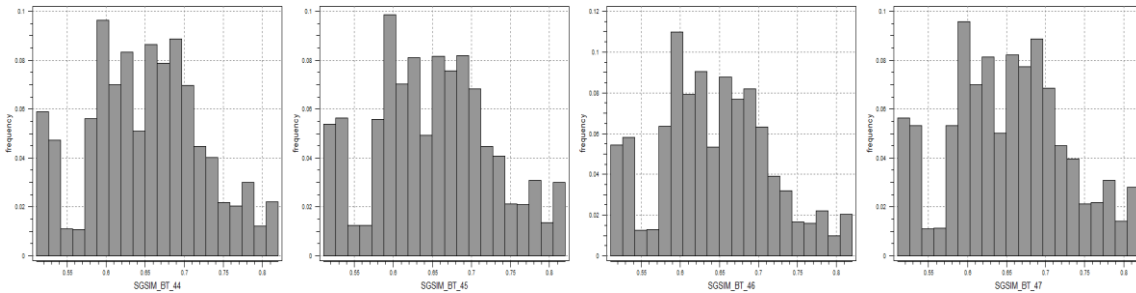


Figure 51: Histograms – Sim_44, Sim_45, Sim_46, Sim_47

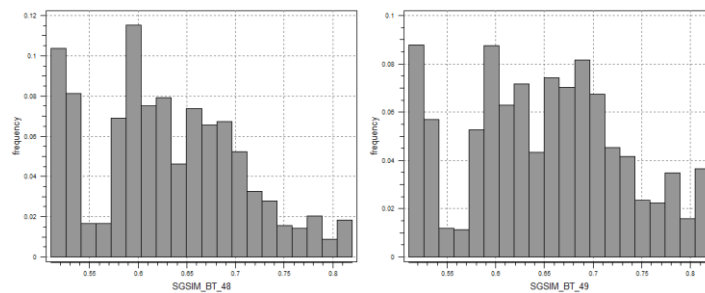


Figure 52: Histograms – Sim_48, Sim_49

The variability can also be compared using the statistical information of the simulations from Table 14 to Table 23.

Table 14: Statistical Parameters – Sim_0, Sim_1, Sim_2, Sim_3, Sim_4

Statistical Parameter	Sim_0	Sim_1	Sim_2	Sim_3	Sim_4
Mean grade (Cu %)	0.649	0.656	0.643	0.639	0.658
Variance	0.008	0.005	0.006	0.006	0.006
Minimum	0.511	0.511	0.511	0.511	0.510
Lower quartile	0.586	0.602	0.589	0.588	0.599
Median	0.650	0.660	0.643	0.629	0.661
Upper quartile	0.710	0.699	0.696	0.693	0.709
Maximum	0.820	0.820	0.820	0.820	0.820

Table 15: Statistical Parameters – Sim_5, Sim_6, Sim_7, Sim_8, Sim_9

Statistical Parameter	Sim_5	Sim_6	Sim_7	Sim_8	Sim_9
Mean grade (Cu %)	0.645	0.652	0.643	0.638	0.640
Variance	0.006	0.007	0.006	0.006	0.005
Minimum	0.511	0.510	0.511	0.511	0.511
Lower quartile	0.593	0.594	0.592	0.588	0.592
Median	0.642	0.652	0.641	0.630	0.636
Upper quartile	0.695	0.703	0.694	0.692	0.692
Maximum	0.820	0.820	0.820	0.820	0.820

Table 16: Statistical Parameters – Sim_10, Sim_11, Sim_12, Sim_13, Sim_14

Statistical Parameter	Sim_10	Sim_11	Sim_12	Sim_13	Sim_14
Mean grade (Cu %)	0.657	0.642	0.652	0.636	0.633
Variance	0.006	0.005	0.006	0.006	0.006
Minimum	0.511	0.511	0.511	0.511	0.511
Lower quartile	0.600	0.592	0.597	0.586	0.583
Median	0.661	0.637	0.654	0.630	0.627
Upper quartile	0.704	0.693	0.699	0.692	0.691
Maximum	0.820	0.820	0.820	0.820	0.820

Table 17: Statistical Parameters – Sim_15, Sim_16, Sim_17, Sim_18, Sim_19

Statistical Parameter	Sim_15	Sim_16	Sim_17	Sim_18	Sim_19
Mean grade (Cu %)	0.651	0.643	0.640	0.639	0.654
Variance	0.005	0.005	0.005	0.005	0.006
Minimum	0.511	0.510	0.511	0.510	0.511
Lower quartile	0.599	0.593	0.592	0.592	0.598
Median	0.654	0.640	0.633	0.635	0.659
Upper quartile	0.697	0.694	0.692	0.691	0.701
Maximum	0.820	0.820	0.820	0.820	0.820

Table 18: Statistical Parameters – Sim_20, Sim_21, Sim_22, Sim_23, Sim_24

Statistical Parameter	Sim_20	Sim_21	Sim_22	Sim_23	Sim_24
Mean grade (Cu %)	0.664	0.646	0.649	0.647	0.651
Variance	0.007	0.006	0.005	0.005	0.005
Minimum	0.511	0.511	0.511	0.511	0.511
Lower quartile	0.603	0.590	0.598	0.597	0.598
Median	0.664	0.645	0.650	0.647	0.653
Upper quartile	0.718	0.698	0.696	0.695	0.698
Maximum	0.820	0.820	0.820	0.820	0.820

Table 19: Statistical Parameters – Sim_25, Sim_26, Sim_27, Sim_28, Sim_29

Statistical Parameter	Sim_25	Sim_26	Sim_27	Sim_28	Sim_29
Mean grade (Cu %)	0.651	0.662	0.635	0.649	0.649
Variance	0.005	0.007	0.006	0.006	0.006
Minimum	0.511	0.511	0.511	0.511	0.511
Lower quartile	0.602	0.601	0.585	0.593	0.595
Median	0.655	0.662	0.627	0.647	0.649
Upper quartile	0.696	0.714	0.691	0.699	0.698
Maximum	0.820	0.820	0.820	0.820	0.820

Table 20: Statistical Parameters – Sim_30, Sim_31, Sim_32, Sim_33, Sim_34

Statistical Parameter	Sim_30	Sim_31	Sim_32	Sim_33	Sim_34
Mean grade (Cu %)	0.646	0.631	0.650	0.651	0.648
Variance	0.005	0.005	0.006	0.006	0.005
Minimum	0.510	0.511	0.510	0.511	0.511
Lower quartile	0.595	0.585	0.595	0.596	0.598
Median	0.648	0.626	0.649	0.656	0.649
Upper quartile	0.695	0.685	0.698	0.699	0.695
Maximum	0.820	0.820	0.820	0.820	0.820

Table 21: Statistical Parameters – Sim_35, Sim_36, Sim_37, Sim_38, Sim_39

Statistical Parameter	Sim_35	Sim_36	Sim_37	Sim_38	Sim_39
Mean grade (Cu %)	0.643	0.646	0.640	0.636	0.623
Variance	0.005	0.006	0.005	0.007	0.005
Minimum	0.511	0.511	0.510	0.510	0.511
Lower quartile	0.595	0.592	0.591	0.582	0.574
Median	0.641	0.644	0.637	0.626	0.618
Upper quartile	0.692	0.697	0.693	0.694	0.674
Maximum	0.820	0.820	0.820	0.820	0.820

Table 22: Statistical Parameters – Sim_40, Sim_41, Sim_42, Sim_43, Sim_44

Statistical Parameter	Sim_40	Sim_41	Sim_42	Sim_43	Sim_44
Mean grade (Cu %)	0.649	0.658	0.656	0.655	0.651
Variance	0.006	0.007	0.006	0.006	0.005
Minimum	0.510	0.511	0.511	0.511	0.511
Lower quartile	0.594	0.597	0.599	0.600	0.599
Median	0.650	0.657	0.661	0.661	0.655
Upper quartile	0.699	0.709	0.704	0.702	0.698
Maximum	0.820	0.820	0.820	0.820	0.820

Table 23: Statistical Parameters – Sim_45, Sim_46, Sim_47, Sim_48, Sim_49

Statistical Parameter	Sim_45	Sim_46	Sim_47	Sim_48	Sim_49
Mean grade (Cu %)	0.651	0.644	0.652	0.630	0.651
Variance	0.006	0.005	0.006	0.006	0.007
Minimum	0.511	0.511	0.511	0.511	0.511
Lower quartile	0.597	0.595	0.598	0.583	0.593
Median	0.653	0.640	0.655	0.623	0.655
Upper quartile	0.699	0.693	0.698	0.684	0.702
Maximum	0.820	0.820	0.820	0.820	0.820

4.4.2 Short-Term Block Model

Following the same procedure as with the estimation method, the point grid model should be average to a point grid of $1.0 \times 1.0 \text{ m}^2$ for each realization. Using the averaged point grid model as hard data, the ordinary kriging is performed in a new grid in SMU size, with a cell size of $10 \times 10 \text{ m}^2$. The outcome will be a set of 50 equiprobable short-term simulated block models.

4.4.3 Medium-Term Block Model

For this case, the same grid configuration is also maintained as in the case of ordinary kriging and following the steps described in Chapter 3.8, the medium-term block model is created for each simulation.

4.5 Grade-Tonnage Curve

The grade-tonnage curve in the stochastic simulation method allows to evaluate the deviation in different equiprobable block models. Figure 53 represents the Block Model grade-tonnage curve of all 50 realizations and also the Ordinary Kriging estimation for different cut-off grades.

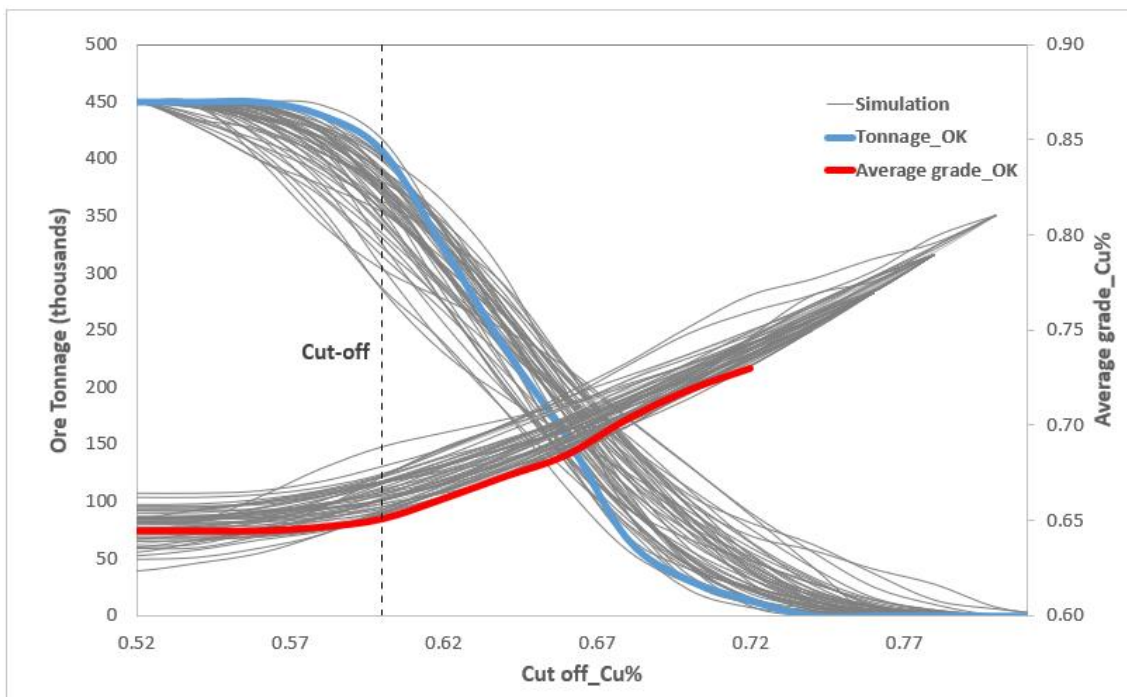


Figure 53: Grade-tonnage curve SGSIM vs OK

4.5.1 Tonnage Distribution

Considering that the cut-off of this project is 0.60%, the distribution of the ore tonnage, schematized in Figure 54, reveals that the Ordinary Kriging estimation tonnage has one of the highest values from all realizations.

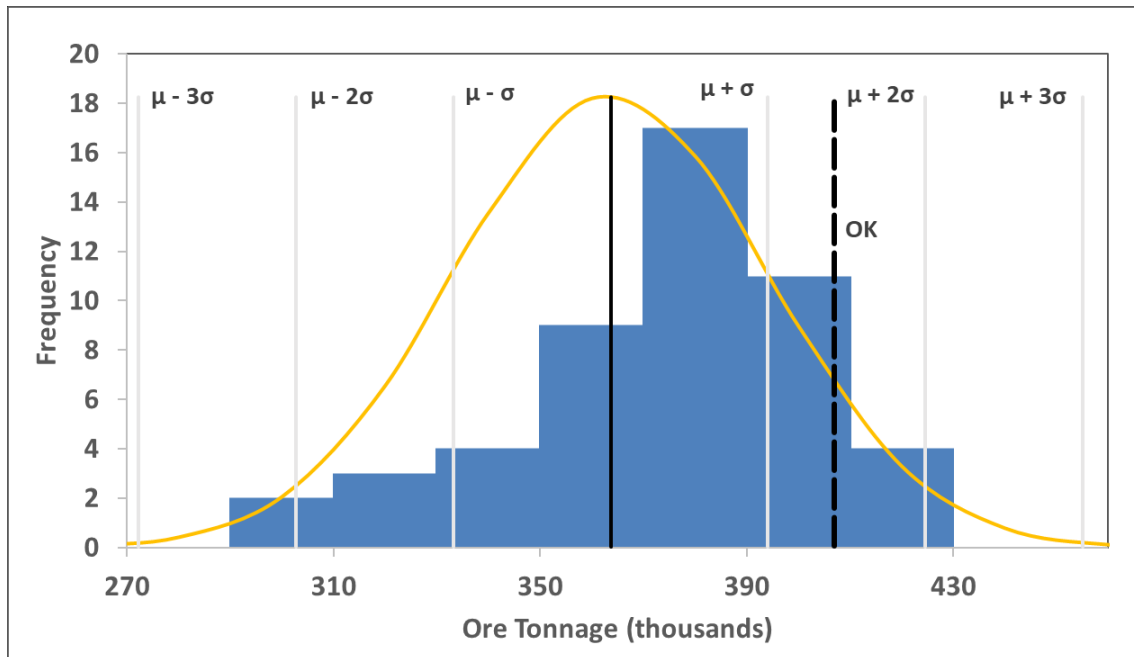


Figure 54: Ore reserve distribution for cut-off of 0.60%

4.5.2 Metal Content Distribution

The geological risk is referred to the grade uncertainty. In this section, the ore grade is measured on its probability of occurrence. Therefore, the metallic content is essential to compare and analyse the uncertainty in grade, as can be seen in Figure 55.

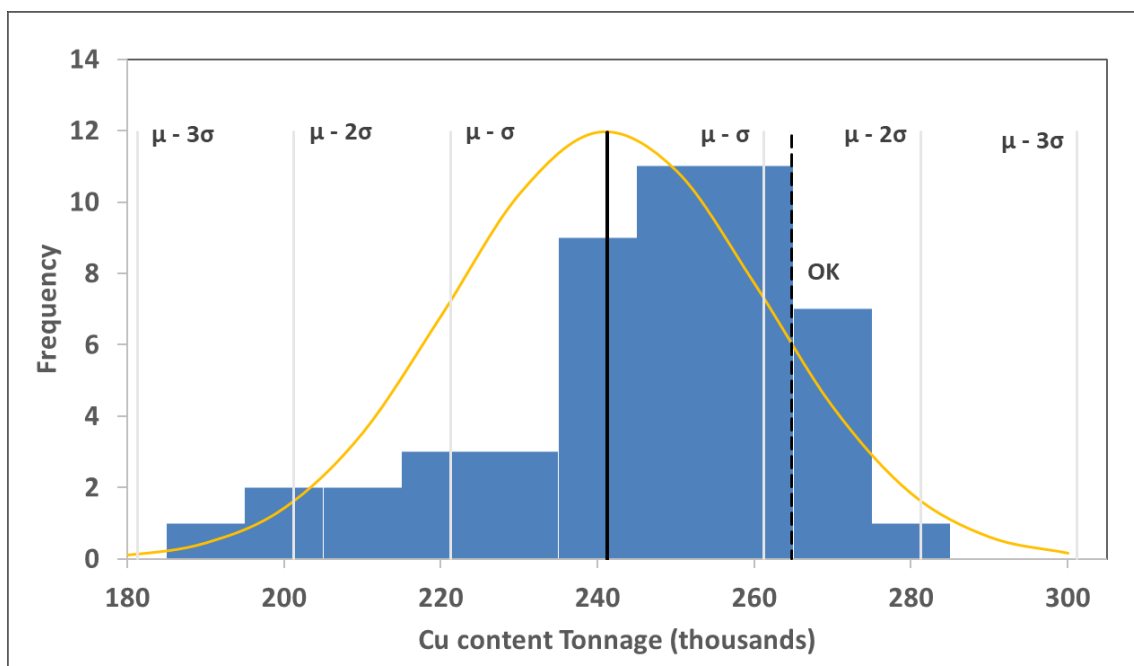


Figure 55: Metallic Content distribution for cut-off of 0.60%

In order to select the best possible solution, the simulations must be analysed considering the highest economical value (NPV) with the lowest or, at least, tolerable geological risk.

To be able to quantify the Net Present Value It is required to develop the sequence of mining in a defined period per each of the block models obtained.

4.6 Mine Planning Scheduling

In this section, the scheduling design will be carried out using the Genetic Algorithm modified for room and pillar, considering the design and operational parameters that can be seen in the following tables. The new optimization approach will allow the development of the mining extraction sequence in a small amount of time, which is a key factor because it is required to process 50 block models.

Table 24: Design Parameters

Parameter	Value
Grid size, m	300x300
Short-term block dimension, m	10x10x2*
Number of short-term blocks, units	900
Short-term block volume, m ³	200
Medium-term block dimension, m	60x60x2*
Number of medium-term blocks, units	25
Medium-term block volume, m ³	7200
Density, t/m ³	2.5
Pillar volume, block %	16

*It is considered that the mining is already in breasting with 2 meters of bench

Table 25: Operational Parameters Short-term

Parameter	Value
Short-term period	Days
Daily production, tn	1,680
Cut-off	0.60%
Number of short-term blocks per day, units	4

Table 26: Operational Parameters Medium-term

Parameter	Value
Medium-term period, days	45
Period production, tn	75,600
Cut-off	0.60%
Number of Medium-term blocks per period, units	5

4.6.1 Medium-Term Scheduling

It will develop a mining sequence per each simulation in medium-term period, following the parameters in Table 26 and connecting them to the optimization approach:

Table 24 indicates that because of the grid size of 300x300, the Lv. 5020 is composed of 25 medium-term blocks.

From the conditions of Table 25 and Table 26, each medium-term block is mined in 9 days, so there must be mined 5 blocks to achieve the production requirement.

In Figure 56 the parameters needed to develop the GA in short-term are shown. The number of bits per chromosome is 25 because this simulates the sequence of all blocks in the grid.

It is considered that, due to the size of the sample, with an initial population of 1000 chromosomes, the number of iterations of 100, a crossover ratio of 90% and a mutation ratio of 5%, correct sequencing can be achieved in a short period. The time elapsed per run was 90 seconds.

```
n_bits=25
n_pop=1000
n_iter=100
r_cross= 0.9
r_mut= 1.25 / float(n_bits)
cut_off = 0.6
n_blocks = 5
```

Figure 56: GA medium-term parameters

Each block has assigned a position value, the scheduling, in general, locates every block in a different position aiming to reach the minimum penalty. The minimum penalty obtained for the OK block model was the following set of values, which are schematized in Figure 57

OK sequence: [0, 3, 5, 8, 6, 7, 11, 13, 18, 12, 10, 4, 15, 14, 9, 17, 2, 19, 1, 20, 21, 23, 24, 16, 22]

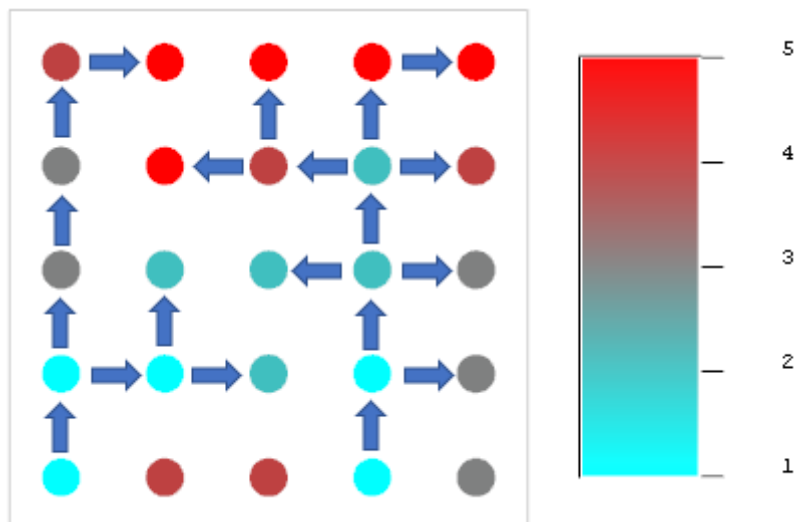


Figure 57: OK Medium-term Sequence – Lv. 5020

This same procedure is developed for each of the 50 simulations. The outcomes will allow to schematize the medium-term mining sequence per period per simulation. This information together with the design, operative and economic parameters, will allow to calculate the NPV.

Table 27: Economic Parameters

Parameter	Value
Mining cost, USD/t	15.73
Ore processing cost, USD/t	15.04
Overhead cost, USD/t	6.59
Ore recovery, %	90.2
Smelting and refining cost USD/t-metal	80
Discount rate, % per year	8
Discount rate, % per Period (45 days)	0.97
Cold Price, Cu USD/t-metal	7,000
Cash-cost \$/tn ore	37.36
Cut-off, %	0.60

Figure 58 and Figure 59 show the range of financial outcomes obtained per simulation. It is appreciated that the Ordinary Kriging value is located in a central position within the range of conditionally simulated realizations.

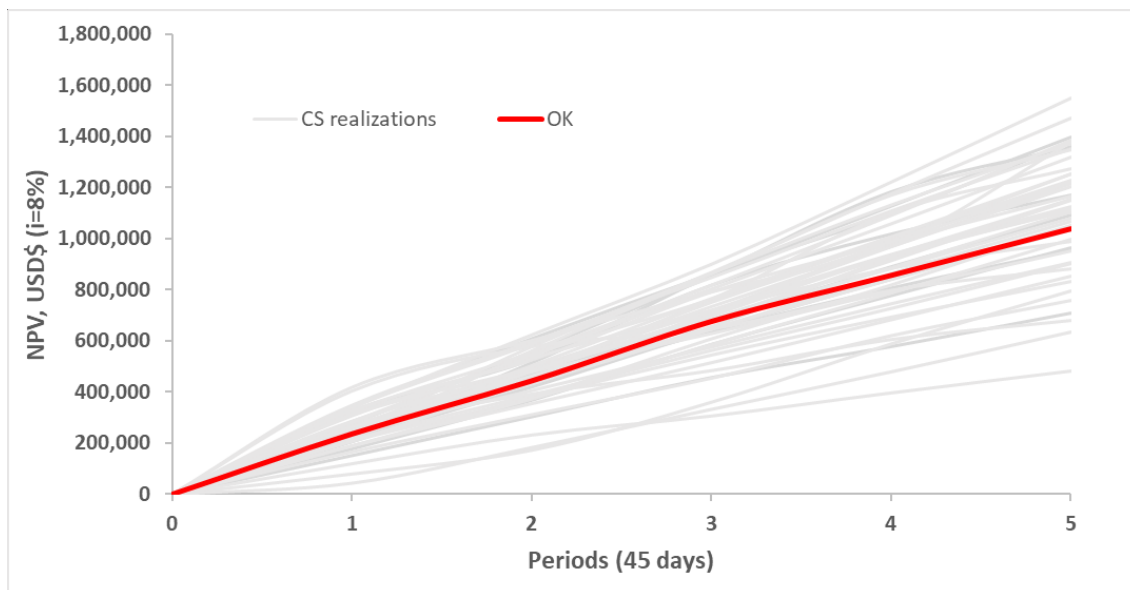


Figure 58: NPV for OK vs SGSIM

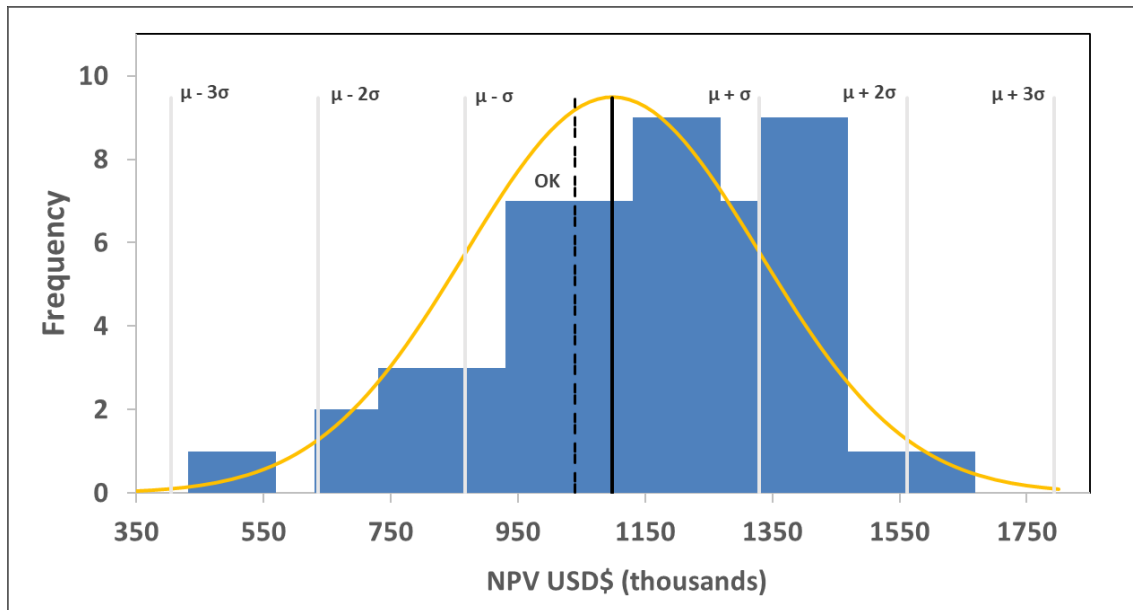


Figure 59: NPV distribution from simulated realizations and OK estimation

4.7 Risk-Benefit Valuation

Among all the stochastic conditioned simulations it is required to select the block model that presents the best valuation following the equation (61). For this reason, it is required to value the economic potential, represented by the NPV, and the geological risk, represented by the metallic content obtained from the grade and tonnage.

4.7.1 NPV Valuation

The application of equation (63) is executed in each of the simulated block models as well as in the OK block model. It is important to mention that the values with higher NPV will be the ones with a higher valuation.

Figure 60 shows the distribution of the NPV valuations among the block models developed reflecting their economic potential. From the graph, the simulations with higher and lower potential are the numbers 26 and 39 respectively. It is important to mention that the OK model has a valuation of 47% which according to Table 6 is classified as “Moderate” risk.

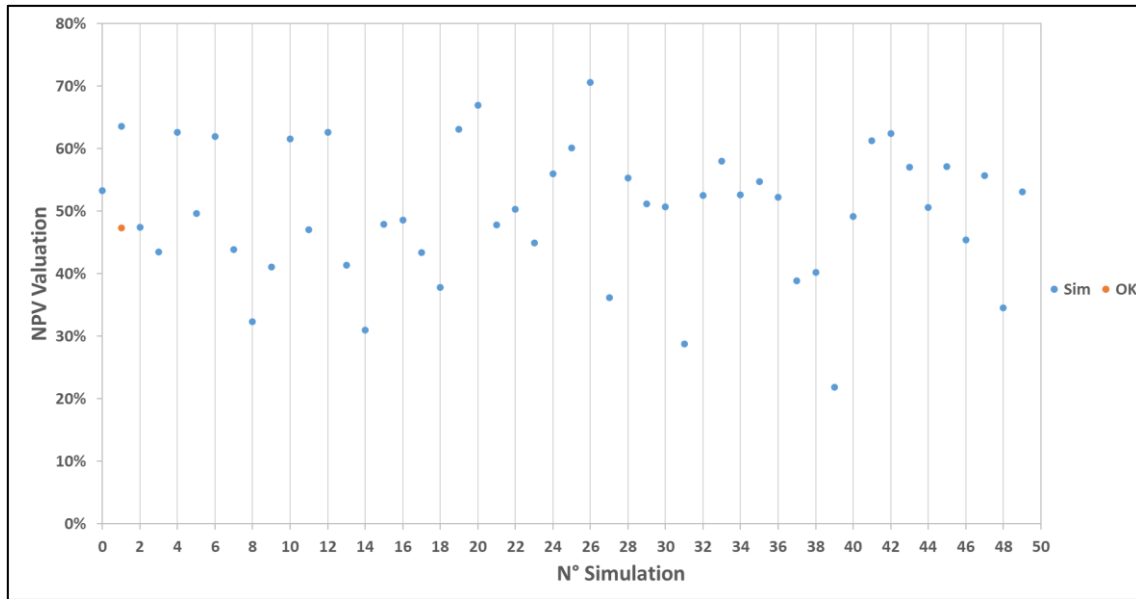


Figure 60: NPV Valuation

4.7.2 Grade Risk Valuation

The grade risk valuation is measured using the distribution of the metallic content. In this case, as farther from the mean as lower the valuation. This first valuation is penalized with a Correction Factor directly linked to the gaussian function properties. From Table 7, Table 28 is developed, which is applied to the case study.

Table 28: Correction Factor for Geological Risk - Applied

Type	Distribution	Range	Correction Factor
Very High	$> \mu - 3\sigma$	181	4.40%
	$< \mu + 3\sigma$	301	
High	$> \mu - 2\sigma$	201	27.20%
	$< \mu + 2\sigma$	281	
Intermediate	$> \mu - \sigma$	221	30.00%
	$< \mu + \sigma$	261	
Low	$> \mu - 0.5\sigma$	231	38.20%
	$< \mu + 0.5\sigma$	251	

The application of this formula allows to recognize that the value with higher valuation has lower grade uncertainty. Figure 61 reveals that there are many simulations in the range of 35% and 40% of valuations, which are the higher range. Nevertheless, the OK estimation is located in a lower section with a value of 24.6% which classify as “High” risk.

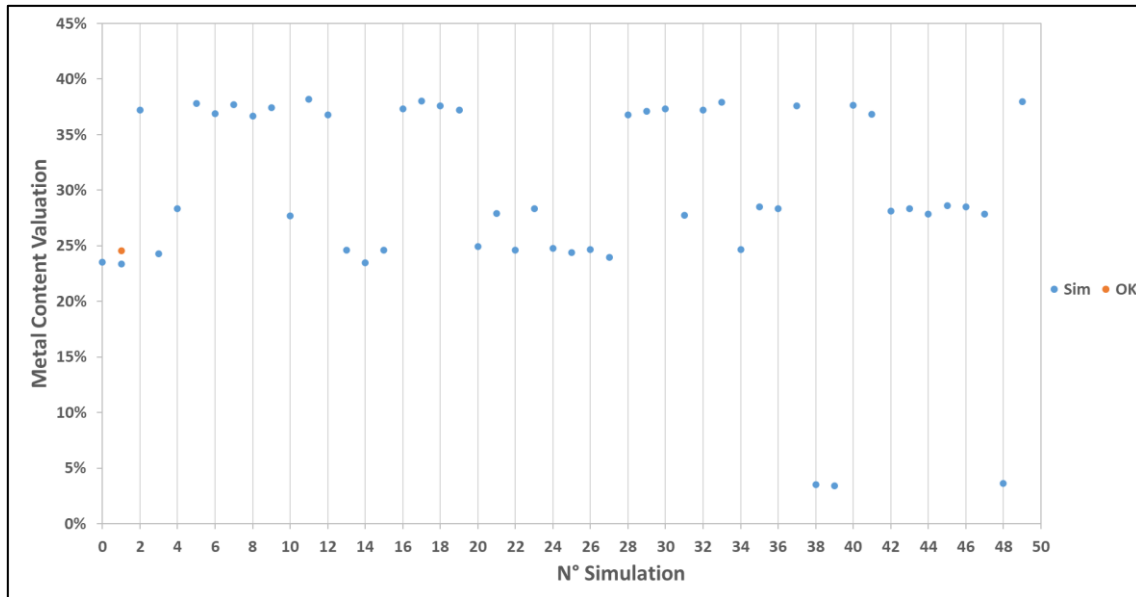


Figure 61: Grade Risk Valuation

The Risk-Benefit Valuation is calculated using equation (61). Figure 62 details the distributions of the results and shows the simulation with the valuation according to the parameters given. In this case, Simulation 19 has the best value with 23.5% of valuation among the 50 realizations and the OK.

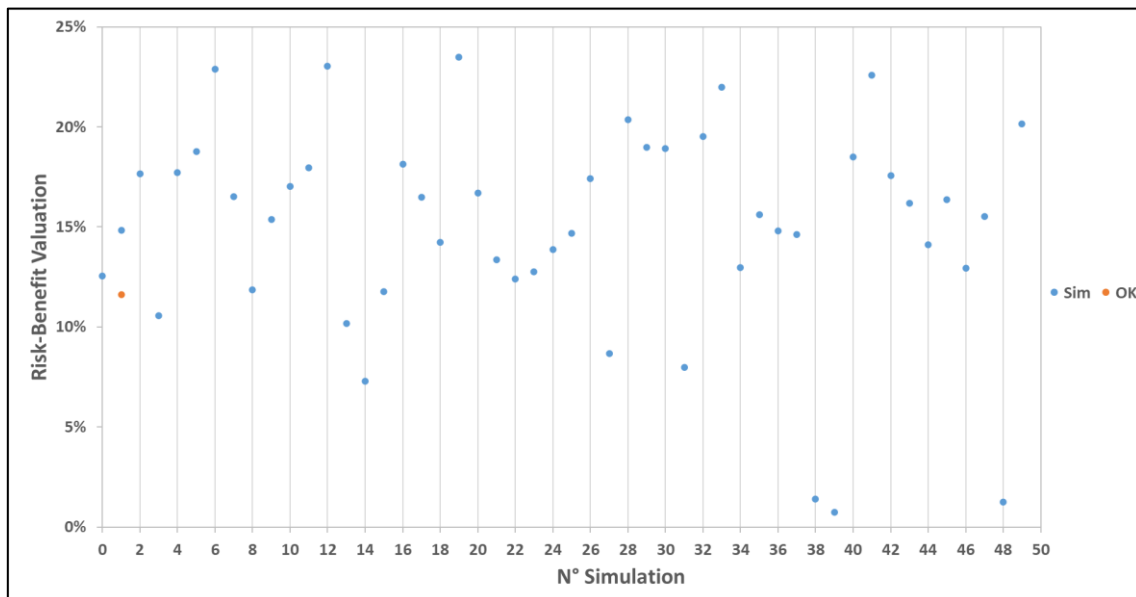


Figure 62: Risk-Benefit Valuation

4.7.3 Risk-Benefit Matrix

Using Table 8 the values obtained in the Risk-Benefit valuation are inserted according to its classification in the 2 aspects previously explained.

There are 4 simulations with the classification of “High” NPV with “Low” Geological risk, and among these block models, simulation 19 is the one with better valuation. On the other hand, the OK estimation is located in the classification of “Moderate” NPV with “High” Geological Risk, which reflects that this estimation has a high probability of not being achieved in reality.

Table 29: Risk Matrix - Applied

Geological Risk			NPV							
			Minimum		Moderate		High		Maximum	
Type	Distribution	Correction Factor	0%	40%	40%	60%	60%	80%	80%	100%
Very High	$> \mu - 3\sigma$ $< \mu + 3\sigma$	4.40%	39, 48		38					
High	$> \mu - 2\sigma$ $< \mu + 2\sigma$	27.20%	14, 27		OK, 0, 3, 13, 15, 22, 24, 34		1, 20, 25, 26			
Intermediate	$> \mu - \sigma$ $< \mu + \sigma$	30.00%	31		21, 23, 35, 36, 43, 44, 45, 46, 47		4, 10, 42			
Low	$> \mu - 0.5\sigma$ $< \mu + 0.5\sigma$	38.20%	8, 18, 37		2, 5, 7, 9, 11, 16, 17, 28, 29, 30, 32, 33, 40,		6, 12, 19, 41			

Depending on the risk level that the company is willing to take, different simulations can be chosen. For the study case, it will be chosen the simulation that has the potential to generate higher economic benefits at lower geological risk is the one with the highest risk-benefit valuation, which is simulation 19.

4.8 Short-Term Planning Scheduling

Once the best medium-term block model is selected, the short-term planning can be developed. For this, it is required first to execute the new optimization approach in short-term size, following the operational and design parameters for short-term previously explained in Table 24 and Table 25. The short-term scheduling must follow the sequence obtained from the medium-term scheduling of Simulation 19.

For Sim19, the scheduling developed using the GA delivering as a result the following sequence, which is schematize in Figure 63:

[0, 5, 3, 1, 2, 7, 4, 12, 17, 9, 18, 8, 23, 10, 6, 19, 15, 13, 22, 20, 11, 16, 21, 14, 24]

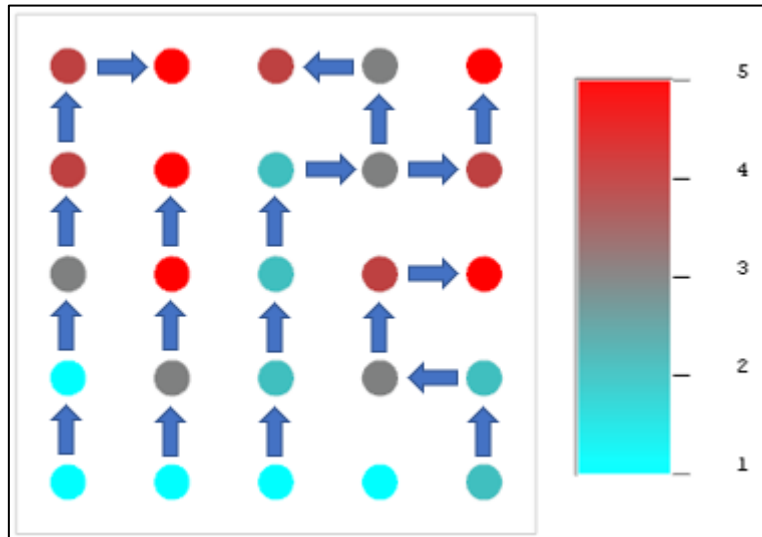


Figure 63: Sim19_Medium-term Sequence

The requirements for the optimization in short-term consider some different parameters than in medium-term. For short-term, the number of bits is 36, because there are 36 blocks in each medium-term block. Because of the processing requirement of 1,680 tn/day, it will be mandatory the extraction of 4 blocks per day with an average grade of 0.6%.

The stochastic algorithm considers an initial population of 1000 chromosomes, a number of iterations of 200, a crossover ratio of 90% and a mutation ratio of 5%, correct sequencing can be achieved in a short period. The time elapsed per run was 200 seconds.

Following the medium-term sequence, the GA modified for “room and pillar” is executed in each medium-term block. Considering that the first block in the medium-term simulation was zero (“0”), this position will be the starting block to initiate the short-term sequence. The optimization provides the following possible solution for the first block in location “0” of the Medium-term schedule of simulation 19 which is schematized in Figure 64:

[0, 6, 12, 18, 24, 7, 5, 25, 26, 27, 19, 31, 20, 8, 30, 9, 28, 4, 13, 10, 11, 14, 3, 17, 15, 34, 1, 16, 21, 32, 35, 2, 23, 29, 22, 33]

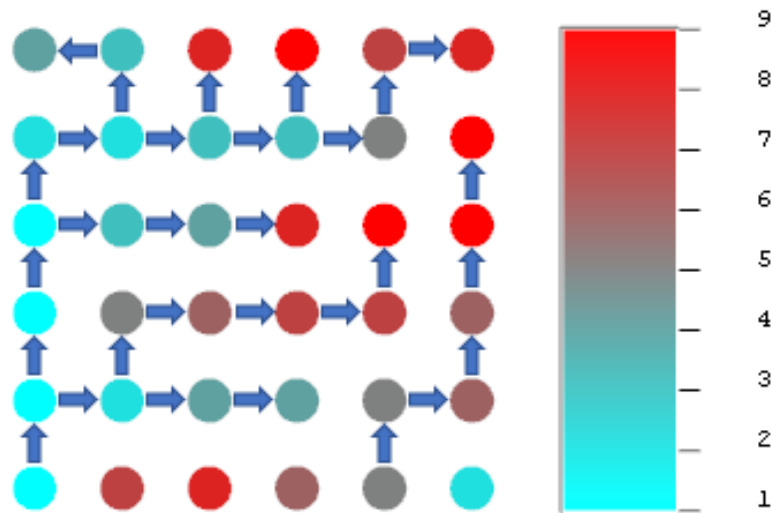


Figure 64: Sim19_Short-term Sequence for Block 0

After running the Genetic Algorithm in each medium-term block, the first short-term scheduling will be obtained, which will be constantly updated as the mining operation progresses. Following the sequence developed, the level 5020 will be mined in 225 days, with a production ratio of 4 blocks per day and a cut-off grade of 0.6%. Figure 65 details the mining sequence in a short-term view, following the same sequence as in medium-term and considering that each medium-term block has 36 short-term blocks.

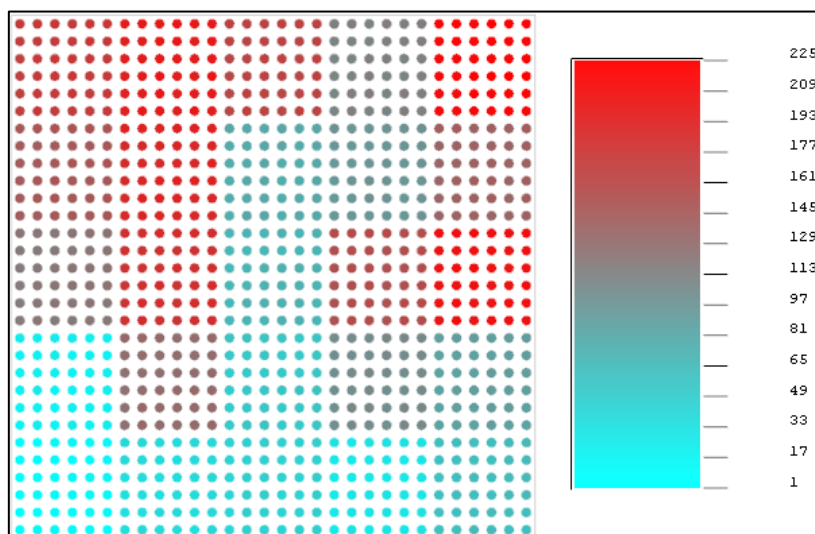


Figure 65: Sim19_Short-term Sequence

4.9 Short-Term Production Program

The short-term planning delivers the daily and monthly production program. The following tables show the mining planning for the first month of operation in which the average grade of 0.67% is achieved, accomplished the condition of 0.60% of cut-off and the production requirement of 1680 tn/day.

Table 30: Production plan for first Block for days 1 - 8

Day	1	2	3	4	5	6	7	8
TM	1,680	1,680	1,680	1,680	1,680	1,680	1,680	1,680
Cu%	0.58	0.66	0.66	0.67	0.65	0.66	0.66	0.66

Table 31: Production plan for first Block for days 9 - 16

Day	9	10	11	12	13	14	15	16
TM	1,680	1,680	1,680	1,680	1,680	1,680	1,680	1,680
Cu%	0.67	0.73	0.75	0.74	0.74	0.74	0.75	0.74

Table 32: Production plan for first Block for days 17 - 24

Day	17	18	19	20	21	22	23	24
TM	1,680	1,680	1,680	1,680	1,680	1,680	1,680	1,680
Cu%	0.74	0.74	0.59	0.61	0.63	0.63	0.65	0.63

Table 33: Production plan for first Block for days 25 - 30

Day	25	26	27	28	29	30	Total
TM	1,680	1,680	1,680	1,680	1,680	1,260	49,980
Cu%	0.64	0.63	0.64	0.68	0.67	0.68	0.67

5 Discussion of Results

- Comparing the values from ordinary kriging estimation against the raw data set of the Lv. 5020, can be appreciated that the smoothing effect is present. The smaller values from the raw data set are overestimated (from 0.52% to 0.57%) and the higher values are underestimated (from 0.82% to 0.74%). Another indicator of the smoothing effect is the diminished of the variance because of the original variance in Lv. 5020 was 0.00605 and after OK it decreased significantly to 0.00130.
- The marginal cut-off is lower than the mine cut-off because the marginal value has sunk cost and is considered that the fixed costs are already paid by the mine cut-off, knowing this any value between 0.52% and 0.60% should be considered as low-grade ore and should be used only for blending purpose.
- The validation of all 50 realizations can be undertaken by comparing the simulated histograms at point scale (Figure 40 to Figure 52) with the original data set histogram in the Lv. 5020 (Figure 23) which are similar.
- The statistical information from Table 14 to Table 23, can also validate the simulations generated. Comparing the variance in Figure 66 is possible to appreciate that the raw data set and the simulations have similar and close values, while, the Kriging has a very low value, close to zero, because of the smoothing effect.
- The realizations obtained prove that all realizations have similar features to the original data and are reasonably accepted.

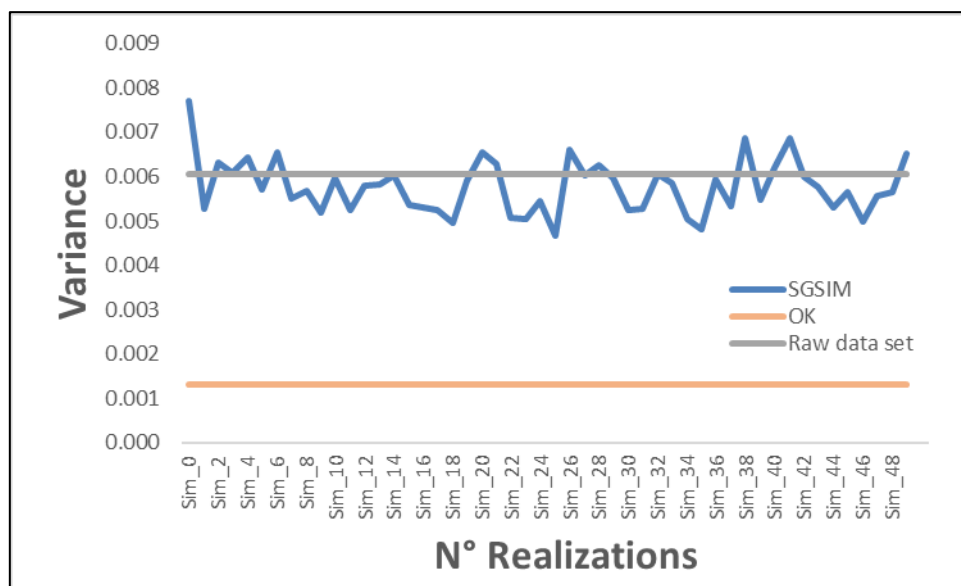


Figure 66: Variance Assessment

- Figure 53 reveals that until the cut-off grade of 0.62% the Ordinary Kriging ore tonnage has a higher value from all over simulations and as the value of the cut-off grade progresses, the tonnage rapidly decreases.
- It is also visible that the average grade of the Ordinary kriging, after the cut-off of 0.58%, has one of the lowest values from all the simulations for each cut-off. Moreover, the maximum average grade obtained for OK is 0.73%, while every simulation always surpasses this value.

- From Figure 53 can be appreciated that the average grade of the OK is the lowest among all realizations with 0.65%. At the same time, in Figure 54 the estimated OK tonnage with 407,000 ore-tones is one of the highest among the simulations because there is only one simulation with a greater tonnage. It is important to mention that this tonnage is located within the range of “ $\mu + \sigma$ ” and “ $\mu + 2\sigma$ ”.
- According to Figure 55 and considering the properties of the gaussian distribution, the estimation of the ordinary kriging of Cu metallic content is located where there is only 31.8% of the simulated values. Having an OK estimation of 264,700 Cu-tons and a mean of 241,200 Cu-tons, it is most likely that the metallic content in the exploitation level 5020 has 10% less than the estimated value.
- The analysis of Figure 58 reveals that the range of financial outcomes from the different simulations is extremely wide, having values between 480K and 1,551K USD\$. Nevertheless, the NPV from the Ordinary Kriging (1,040K USD\$) is located inside the range of “ $\mu \pm \sigma$ ” (639,000 - 1,238,000 USD\$) which contains 68.2% of the values.
- According to the results obtained from Figure 60, 66% of the realizations generate a higher economic value than the Ordinary Kriging.
- The OK NPV valuation of 47% indicates that its potential is less than the average economic potential value from the 50 simulations. This result is logical considering that the OK method produces an underestimation in large values. This statement can be proved using the statistical information from Table 14 to Table 23 and evaluating the “upper quartile”. Figure 67 schematizes the upper quartile assessment, showing that the upper quartile of OK is the lowest among all simulations which indicates that the simulation has more times higher grades, therefore higher economic value.

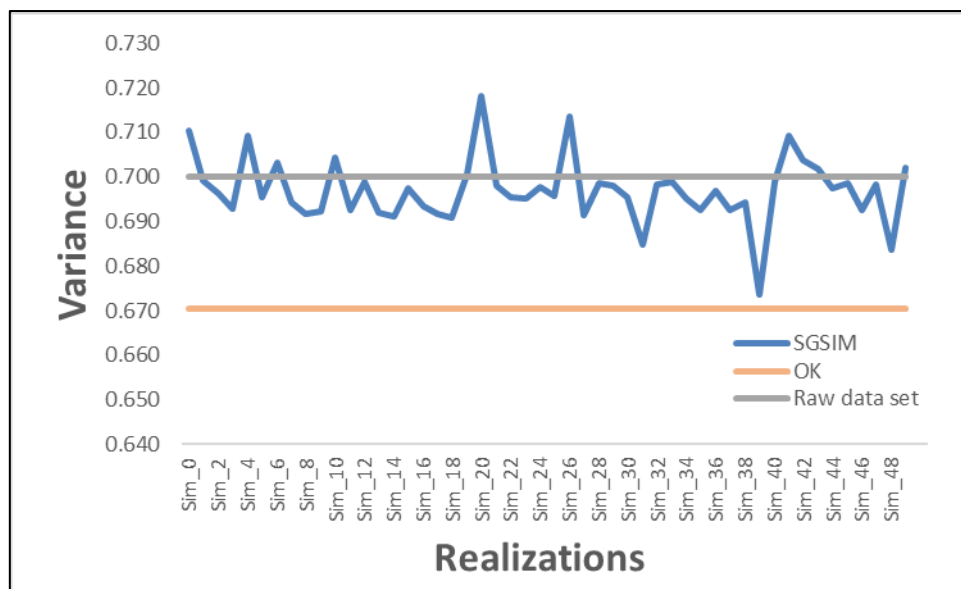


Figure 67: Upper Quartile Assessment

- The outcome of having the OK model with a classification of “High” risk in the grade valuation section is logical, considering that its metal content and tonnage were outside of the gaussian range distribution of “ $\mu \pm \sigma$ ”.

- It is proven that the GA modified for “room and Pillar” is a very useful tool to carry out mining scheduling, since it can successfully develop the algorithm in times less than 90 and 120 seconds for the medium and short-term block models, respectively.
- It was essential to divide the block model in short and medium-term, to reduce the number of blocks per model. If this had not been done, 900 bits per block model would have had to be processed with the genetic algorithm, which would have produced a significant increase in processing time.
- Simulation 38, 39 and 48 have the lowest values with less than 2%, which refers to a risk classification of “Very High”. This is generated since its metal content is between the range “ $\mu - 2\sigma$ ” and “ $\mu - 3\sigma$ ”
- Depending on the risk that the company is willing to take, it could also be possible to consider different simulations that could generate higher NPV with a higher geological risk. On the other hand, a more conservative company could choose a model with the lowest geological risk which in this case is Simulation 11 having a moderate NPV potential.
- It can be appreciated, that 11 simulations have higher NPV than the OK model with the same or lower geological risk.
- The daily production program, shown from Table 30 to Table 33 successfully meet the operative requirements. Day 1 and 19 are the only days in which the grade is less than the cut-off; nevertheless, the average grade of the month is 0.67%.

6 Conclusions

The thesis work proves satisfactorily that it is possible to address the geological uncertainty and develop short-term production programs in a reasonable amount of time, which will be highly beneficial to real mining operations.

When the grade uncertainty is included in the planning process, it will generate more robust mining plans with lower grade deviations, which will generate fewer economic losses and less operational improvisation to meet the requirements of the processing plant.

The limitations that deterministic methods like Kriging have been significant because it only allows having a single and unique estimated block model, making it not possible to develop any geological risk assessment or evaluate the NPV potential. At the same time, it produces the smoothing effect which can lead to under or overestimations in the block model and could produce shortcomings in mining planning especially in reaching short-term targets.

The main drawback of the Sequential Gaussian Simulation is the processing time because it will be conditioned to the computing capacity, the size of the deposit and the number of realizations, which would make the simulation process slow. However, there are more advanced conditioned simulated methods that can overcome the time consumption difficulty.

The Genetic Algorithm modified for “room and pillar” proves to be a powerful tool for mining planning generating suitable schedules in a short time. It is true that, in the study case, the restrictions were simple; however, the algorithm can be adapted to more complex requirements of any mining operation.

The Risk-Benefit matrix proves to be of paramount importance to classify in a practical way every realization and estimation created, allowing to identify the best and worst block models.

Depending on the targets and strategies of the mining operation, it is possible to select a block model different than the one with the highest Risk-Benefit valuation. This decision depends mainly on the level of geological risk that the company is willing to take.

Knowing that if the extracted grade does not meet the expected grade of the block model, there could be a risk that the entire project fails. For this reason, it is of paramount importance to develop a geological risk analysis and be able to assess the accuracy of the modelled information.

The versatility of the genetic algorithm allows updating the sequencing conditions and restrictions in case the laws in the exploitation area are updated. This would allow the short-term plan to be adopted rapidly.

Geological uncertainty is an inherent parameter in every block model; therefore, it is verified that a single deterministic estimation is not sufficient to be able to consider the full potential of the mineral deposit.

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List of Abbreviations

UCS	Uniaxial Compressive Strength
OK	Ordinary Kriging
SGSIM	Sequential Gaussian Simulation
GA	Genetic Algorithm
EDA	Exploratory Data Analysis
SMU	Selective Mining Unit
NPV	Net Present Value
SK	Simple Kriging
pdf	Probability Density Function
cpdf	Conditional Probability Density Function