Application of Gassmann's equation for laboratory data from carbonates from Austria

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Abstract

Gassmann's "fluid substitution" equations belong to the most popular approaches to calculate velocities for rocks saturated with one fluid (1) and substituted with another fluid (2). Due to the limitations, the equations can hardly be used for carbonate samples. Different carbonate samples (limestone and dolomite) from Austria are selected for testing Gassmann's equation and modifications, and compressional and shear wave velocity for dry and saturated samples as well as porosity and density were determined in the laboratory. The next step was the calculation of the compressional and shear wave velocities for brine saturated samples using Gassmann's equation as a first approach. The second approach was to directly use the modulus k_1 calculated from dry measured data rather than the compressional modulus k_{dry} for the dry rock frame, and in the third approach the Lamé parameter λ was used for k_1. λ was also calculated directly from the measured dry data and covers the pure incompressibility. These three approaches were not only tested for the Austrian carbonates, where the porosity is very low, but also for a data set from chalk limestone samples with high porosity. The best results can be observed using k_1 directly. Additionally using k_{dry} leads to an underestimation of the data. There is no difference between using k_1 or λ for the low porous Austrian carbonates. In contrast, the high porous chalk limestone samples show hardly any difference between the k_1 and λ approach for the highest porosities, but a scatter when using λ for the lower porosities. In summary it can be said that Gassmann's equation directly using k_1 from the measured data or λ deliver good results for low and high porous carbonate laboratory data.

Gassmann's "fluid substitution" Gleichungen gehören zu den meist genutzten Anwendungen um Geschwindigkeiten für ein gesättigtes Gestein (Fluid 1) zu berechnen, welches mit einem Fluid (2) ersetzt werden soll. Durch ihre Einschränkungen, kann diese Gleichung kaum für Karbonate angewandt werden. Um diese Gleichungen trotzdem zu testen und um zu versuchen sie zu modifizieren, wurden unterschiedliche Karbonate (Kalkstein und Dolomit) aus Österreich ausgewählt. Es wurden Kompressions- und Scherwelle von trockenen und gesättigten Proben, ebenso wie Porosität und Dichte im Labor bestimmt. Der nächste Schritt war die Berechnung der Kompressions- und Scherwelle mit der Gassmann Gleichung als ersten Ansatz für gesättigte Proben. Für die zweite Methode wurde anstatt des Kompressionsmoduls k_{dry} für das „trockene Gesteinsgerüst" direkt der Modul k_1, aus den Messungen berechnet, verwendet. Der dritte Ansatz ist die Verwendung des Lamé Parameters λ anstelle von k_1. λ wurde ebenfalls direkt aus den Messungen der trockenen Proben berechnet und umfasst die reine Inkompressibilität. Diese drei Ansätze wurden nicht nur für die österreichischen Karbonate mit geringer Porosität, sondern auch für einen Datensatz mit Kalksteinen mit höheren Porositäten angewandt. Die besten Ergebnisse konnten beim direkten Verwenden von k_1 erzielt werden. Bei der zusätzlichen Verwendung von k_{dry} wurden die Daten unterschätzt. Es konnte kein Unterschied zwischen den beiden Ansätzen mit k_1 und λ für die gering porösen Karbonate von Österreich beobachtet werden. Im Gegensatz dazu zeigen die hoch porösen Kalksteinproben kaum einen Unterschied zwischen k_1 und λ, aber eine Datenstreuung für die niedrigeren Porositäten. Zusammengefasst lässt sich sagen, dass die Gassmann Gleichungen mit der direkten Verwendung von k_1 aus den gemessenen Daten oder λ gute Ergebnisse für gering und hoch poröse Karbonate im Labor liefern.

1. Introduction

Seismic properties of carbonate rocks and their dependence on pore fluid saturation are of interest for solving geological problems in Austria because

- Carbonates represent important reservoirs for hydrocarbons, particularly gas detection is subject of application,
- Carbonates with different hydrogeological positions (e.g. dry/water bearing are subject of engineering problems (tunneling etc.))

Gassmann's equation (1951) is the most popular approach to determining velocities at various pore fluid contents. The equation allows the calculation of wave velocities for any pore fluid if velocities are known (measured) at one pore fluid. This is realized with a so called "fluid substitution". Gassmann's theory is frequently applied for the fluid estimation from velocities of sandstone (gas bearing versus water bearing). Due to assumptions and limitations, such as the constant shear modulus or fully interconnected pore space it is often not applicable for carbonates. Many authors discussed the problems, often combined with the observed effect called shear weakening (e.g. Røgen et al., 2005; Sharma et al., 2011, Khazanehdari

and Sothcott, 2003). Carbonate data show a lower shear modulus for saturated samples compared to dry samples. Therefore Gassmann's assumption that the shear modulus for dry and brine saturated data stays the same is not valid anymore. Eberli et al. (2003) note "different pore types cluster in the porosity-velocity field, indicating that scattering at equal porosity is caused by the specific pore type and their resultant elastic property".

Baechle et al. (2005, 2009) present an interpretation of their derived data concerning various pore types. They also observed the shear weakening effect and compared the measured velocities to ones calculated with Gassmann's equation. The calculated velocities are constantly underestimated. Their explanation for the change of the shear modulus is a rock-fluid interaction. Gomez et al. also (2007) published data where Gassmann underpredicts the saturated bulk modulus. Approaches for using Gassmann's equations are, for example, discussed by Assefa et al. (2003), who published compressional and shear wave velocities for dry and saturated limestone. They carried out measurements of vp and vs under in situ conditions for dry and saturated samples and analyzed the data linked to porosity and pore structure. Ciz and Shapiro (2007) presented a generalized Gassmann equation for a solid filled pore space. An improvement for disconnected and partially connected porosity was given by Grechka (2009) and Grochau and Gurevich (2009) tested the Gassmann equation for carbonates.

Gassmann's theory is frequently applied for fluid estimation from velocities of sandstone (gas bearing versus water bearing). Gassmann's assumptions and limitations (Dewar and Pickford, 2001) are:

- that the rock is macroscopically homogeneous and isotropic: this assumption ensures that wavelength > grain and pore size (this is given in most cases of seismic field and laboratory measurements). The statistical isotropic porous material with homogeneous mineral moduli makes no assumptions with respect to any pore geometry.
- that within the interconnected pores there is a fluid pressure equilibrium and no pore pressure gradient as a result of passing waves. Thus, the low frequency allows an equilibration of the pore pressure within the pore space. Therefore Gassmann's equation works best for seismic frequencies (< 100 Hz) and high permeabilities (Mavko et al., 2011).
- that pores are filled with non-viscous, frictionless fluids. This also contributes to pore pressure equilibrium and results in a fluid independent shear modulus of the porous rock.
- that the rock-fluid system is closed (undrained), i.e., no fluid can flow in or out of the considered volume during wave passage.
- that the pore fluid does not interact with the solid material or rock frame. Gassmann's mo-

del does not implement any change of the "rock skeleton or frame modulus" by changing fluids (e.g., softening in case of swelling clay cement by replacement of oil by water with reactive chemical composition or in general as a result of changing surface energy).

- that a passing wave results in the motion (displacement) of the whole rock section, but there is no relative motion between the solid rock skeleton and the fluid. This is given only for zero frequency (static solution); for high frequencies a relative motion can result in dispersion.

The purpose of this paper is to present two approaches for the calculation of water statured velocities from data of dry carbonates, where the measured k1 and Lamés parameter λ is used rather than the dry compressional modulus (kdry) of Gassmann's equation. It is often not possible to measure the saturated velocities, even in the laboratory. Therefore, these approaches can help and further the understanding of elastic properties which starts in the laboratory. The idea of using Lamés parameter was developed through analyzing the data
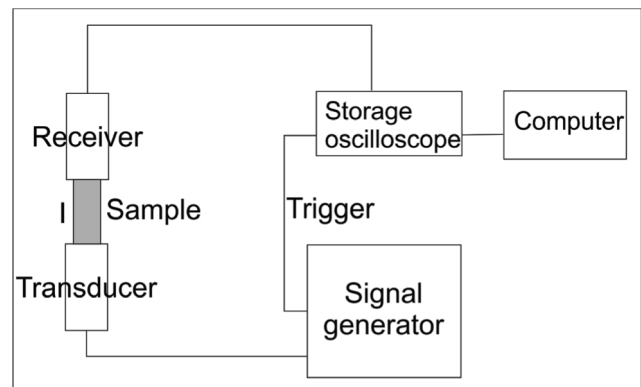

Figure 1: Schematic illustration of the ultrasonic device.

with the new approaches in AVO analyses where the data are interpreted using plots of λ versus μ or λρ versus μρ. This method is mainly applied to discriminate gas and water zones, due to the fact that λ covers pure incompressibility (Russell et al., 2001, Goodway et al., 2010). Measured saturation data are compared with the calculated values. An additional data set from Rogen et al. (2005) was analyzed in the same way. The chalk limestone there has higher porosities.

Rock Type	Grain density	Porosity	$v_{p,dry}$	$v_{p,sat}$	$v_{s,dry}$	$v_{s,sat}$
	g/cm³	%	m/s	m/s	m/s	m/s
"Dachstein"-limestone	2.73	2.33	5799	6143	3101	2873
Dolomite	2.85	3.89	5535	5540	3793	2940
"Haupt"-dolomite	2.84	3.81	5592	6128	3061	2815
Limestone	2.72	2.07	5422	5900	3119	2749
Mix Limestone	2.73	7.55	4536	4750	2818	1767
"Schoeckel"-limestone	2.73	1.09	5783	6231	3622	3074
"Wetterstein"-dolomite	2.84	4.86	4996	5970	2767	2667

Table 1: Overview of the mean values of the singular rock types.

2. Method

2.1 Samples

Various carbonate types from Austria with low porosity are used, namely the "Wetterstein"-dolomite, "Dachstein"-limestone, "Schoeckel"-limestone, and "Haupt"-dolomite. These carbonates were used for a newly developed rock physics template (Gegenhuber and Pupos, 2015). The rock physics template enables the discrimination of the samples based on their pore fluid. Table 1 gives an overview of the mean values for the singular rock types in the following figures.

Most of the samples are from outcrops, fresh with no visible alterations. The samples were selected using geological maps and classified according to these descriptions. "Wetterstein"-dolomite represents part of the middle and upper Triassic section, and samples come from a quarry in Rohrbach (Upper Austria). The "Dachstein"-limestone from a quarry in Ebensee (Upper Austria) belongs to the upper Triassic of the Northern Calcareous Alps. "Haupt"-dolomite samples (upper Triassic) are from a quarry in Gaaden/Mödling (Lower Austria). The "Schoeckel"-limestone samples (Paleozoic in age) are from a project with the Austrian Geological Survey (GBA) and are partially from drilling cores of the GBA and from stone pits in lower Styria. A second data set presented by Røgen et al. (2005) is used to verify the results. These chalk limestone samples have higher porosities than the ones from Austria and therefore offer a good possibility for testing the methods.

2.2 Measuring Method

Plugs (2.5cm diameter and a length between 2.0-2.2cm) were used for the laboratory measurements. Grain density was determined with a helium-pycnometer (Quantachrome, England) on the dried samples (one night 105°C) and the effective po-
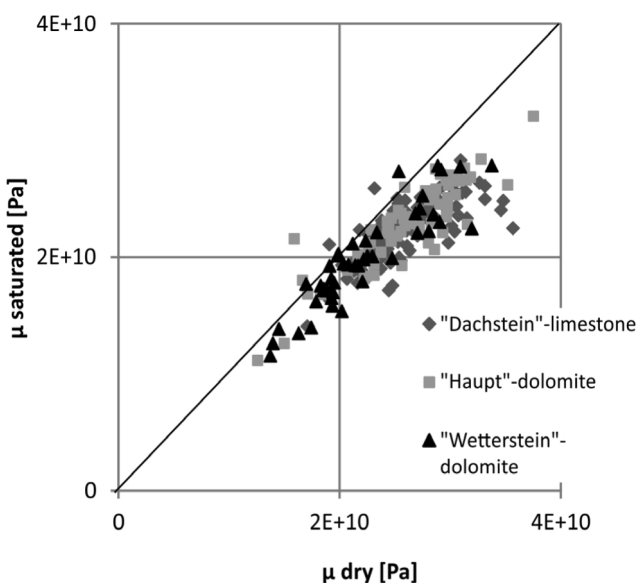
Figure 2: Shear modulus of the saturated samples versus the shear modulus of the dry samples for the three different formations, dark grey: "Dachstein"-limestone, light grey: "Haupt"-dolomite, black: "Wetterstein"-dolomite.

rosity (=connected pore space) was calculated using the bulk density and the derived grain density. The velocities were determined with an ultrasonic device (Figure 1), which is a bench top instrument, on core samples dried overnight and saturated with brine (1000ppm NaCl). The sample was fixed between transmitter and receiver (Geotron Elektronik, Germany) with a contact agent and an axial pressure of 2bar was applied, for a better coupling between the sample and the transducer. An impulse (80kHz) was sent to the transducer resulting in a mechanical pulse that travels through the sample in the form of a wave. The arriving signal was visualized on the computer screen with a storage oscilloscope. All measurements were corrected for dead time. As the signal of such a measurement shows similar characteristics to seismograms, the onset of v_p and v_s, respectively are detected with the Akaike Information Criterion Picker (AIC) (Gegenhuber and Steiner-Luckabauer, 2012).

2.3 Calculations

Three calculations were carried out for the two data sets. The first approach for our carbonate data set was to test the Gassmann equation, the way it is typically used:

$$M = v_p^2 * \rho \tag{1}$$

$$\mu = v_s^2 * \rho \tag{2}$$

$$k_1 = M - \frac{4}{3}\mu \tag{3}$$

$$k_{dry} = \frac{k_1*(1-\Phi)+k_1*\frac{k_s}{k_{fl}}*\Phi-k_s}{\frac{k_1}{k_s}+\frac{k_s}{k_{fl}}*\Phi-(1+\Phi)} \tag{4}$$

$$k_{sat} = k_{dry} + \frac{(1-\frac{k_{dry}}{k_s})^2}{\frac{\Phi}{k_{fl}}+\frac{(1-\Phi)}{k_s}-\frac{k_{dry}}{k_s^2}} \tag{5}$$

$$v_{p,sat} = \sqrt{\frac{k_{sat}+\frac{4}{3}\mu}{\rho}} \tag{6}$$

ρ…bulk density (g/cm³)
μ…shear modulus (Pa)
k_1…rock compressional modulus from the measured dry data (Pa)
k_{dry}…calculated dry rock frame compressional modulus (Pa)
k_{sat}…calculated saturated rock compressional modulus (Pa)
k_s…compressional modulus of the matrix (literature value) (Pa)
k_{fl}…compressional modulus of the fluid (literature value) (Pa)
$v_{p,sat}$…calculated saturated compressional wave velocity (m/s)

Equation 4 delivers the dry rock frame modulus of the dry rock frame or skeleton, calculated from the measurements of the dry data. Values of k_s=9.4E10Pa for dolomite and 7.5E10Pa for limestone and k_{fl} for air 1.0E5 and for water 2.2E9Pa were used. Density was calculated directly with the measured effective porosity and grain density. k_1 was used in equation 5

instead of k_{dry} due to the fact that the results were not sufficient. This was also mentioned by Mavko et al. (2011), where it is stated that for laboratory data, where air is present in the samples, k_{dry} does not need to be used due to the fact that the gas in the reservoir has a different compressibility than the air in the laboratory.

$$k_{sat,1} = k_1 + \frac{(1-\frac{k_1}{k_s})^2}{\frac{\Phi}{k_{fl}} + \frac{(1-\Phi)}{k_s} - \frac{k_1}{k_s^2}} \tag{7}$$

$$v_{p1,sat} = \sqrt{\frac{k_{sat1} + \frac{4}{3}\mu}{\rho}} \tag{8}$$

The third approach was to use λ from the measurements instead of k (equation 5), which results in:

$$\lambda_{sat,1} = \lambda_{dry} + \frac{(1-\frac{k_{dry}}{k_s})^2}{\frac{\Phi}{k_{fl}} + \frac{(1-\Phi)}{k_s} - \frac{k_{dry}}{k_s^2}} \tag{9}$$

$$v_{p2,sat} = \sqrt{\frac{\lambda_{sat} + 2\mu}{\rho}} \tag{10}$$

λ stands for the pure incompressibility and covers only the fluid effects. Therefore λ calculated from the measured dry data were used for λ_{dry}. These three approaches are presented, compared and discussed in the following chapter.

3. Results and interpretation

Figure 2 demonstrates the so called shear weakening effect (see also Gegenhuber and Pupos, 2015), figure 3 shows the Austrian carbonate data set. The first plot (Fig. 3a) shows measured compressional wave velocity versus the calculated v_p with the Gassmann equation using k_{dry}. It can be clearly observed that calculated data are completely underestimated. This effect is also stated by other authors (e.g. Røgen et al., 2005 and Sharma et al., 2006) and was therefore expected.

The second approach using k_1 instead of k_{dry} for the Gassmann equations is presented in figure 3b, where again measured versus calculated compressional wave velocity is plotted. It can be observed that the dolomite samples are slightly underestimated but less so than with the original equations, and the limestone shows better results than in the first approach.

The third plot (Fig. 3c) shows the last approach using λ instead of k_{dry}. The last two approaches deliver the same results for this data set. Therefore, it can be verified that using λ instead of k can also be used for low porosity carbonates.

Figure 4 shows the differences between the two approaches to make the results better visible. Therefore, $v_{p,sat,measured}$-$v_{p,calculated}$ was derived for the three approaches. It becomes visible that the Gassmann equation using k_{dry} underestimates the compressional wave velocity for the complete data set
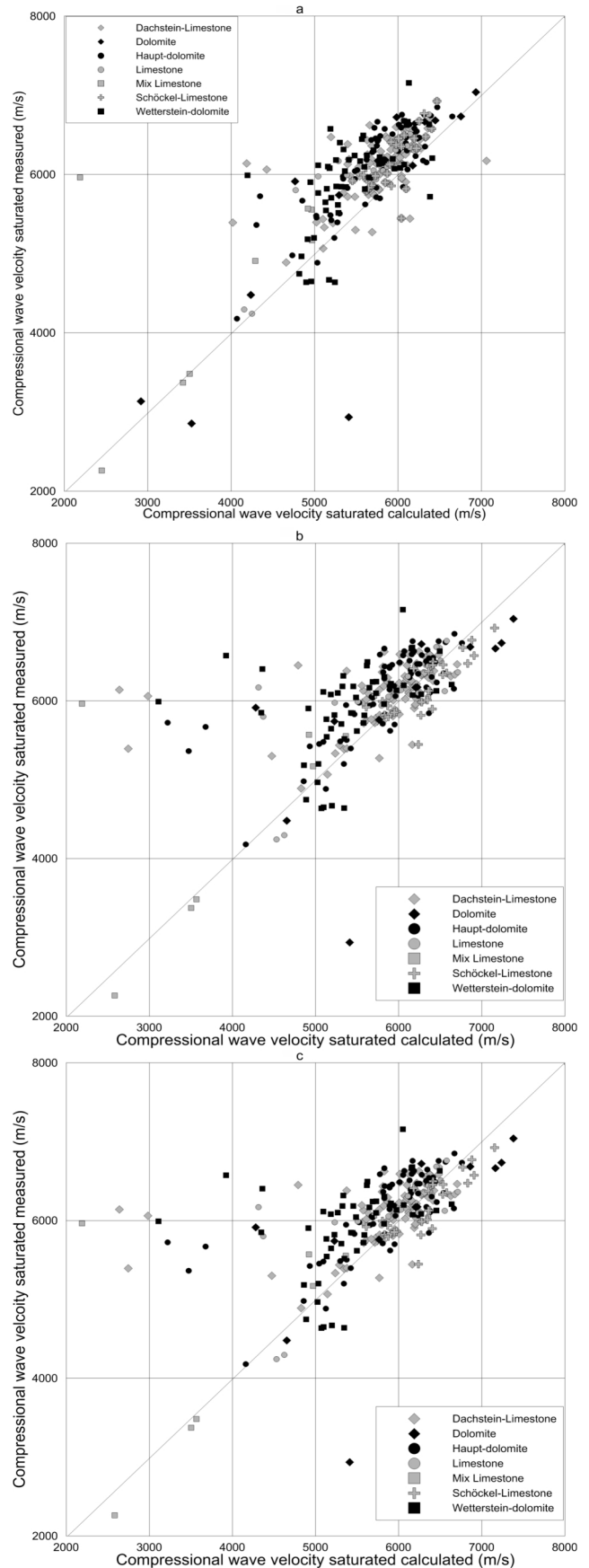
Figure 3: a: measured compressional wave velocity saturated (v_p) versus calculated with the original Gassmann equation; b: measured v_p versus calculated with the Gassmann equation using k_1 instead of k_{dry}. c: measured v_p versus Gassmann using λ, grey: limestone, black: dolomite, line shows $v_{p,sat,calculated} = v_{p,sat,measured}$

(between 0-1000m/s). The porosity is plotted on the y-axis. Data scatter more with increasing porosity. The other two approaches underestimate the dolomite samples less than the first equation while there is a slight overestimation for the limestone. In general the two approaches using k_1 and λ deliver better results. Therefore, the citation by Mavko et al. (2011) that for laboratory data k_{dry} must not be used can be proved here.

A data set presented by Røgen et al. (2005) was used for a comparison and verification of this approach. The chalk limestone samples have higher porosities. Figure 5a shows measured versus calculated saturated compressional wave velocity for the three presented approaches. The Gassmann equation using k_{dry} leads again to a clear underestimation (200-600m/s) of the compressional wave velocity (Fig. 5b). With

the modified approach, data are slightly overestimated but only by about 200m/s. The best results can be observed using k_1. In contrast to the data set with low porosities, a difference between the two approaches (λ und k_1) can be observed. Both show better results than when using k_{dry}, but using λ led to an increased scatter of the data (100m/s) compared with using k_1. The values for the highest porosities show a strong underestimation when using k_{dry} and nearly the same low overestimation using k_1 and λ.

4. Conclusion

This paper presents the results and comparison of various approaches using Gassmann's equation for low and high porous carbonate samples. The first approach calculating k_{dry} for
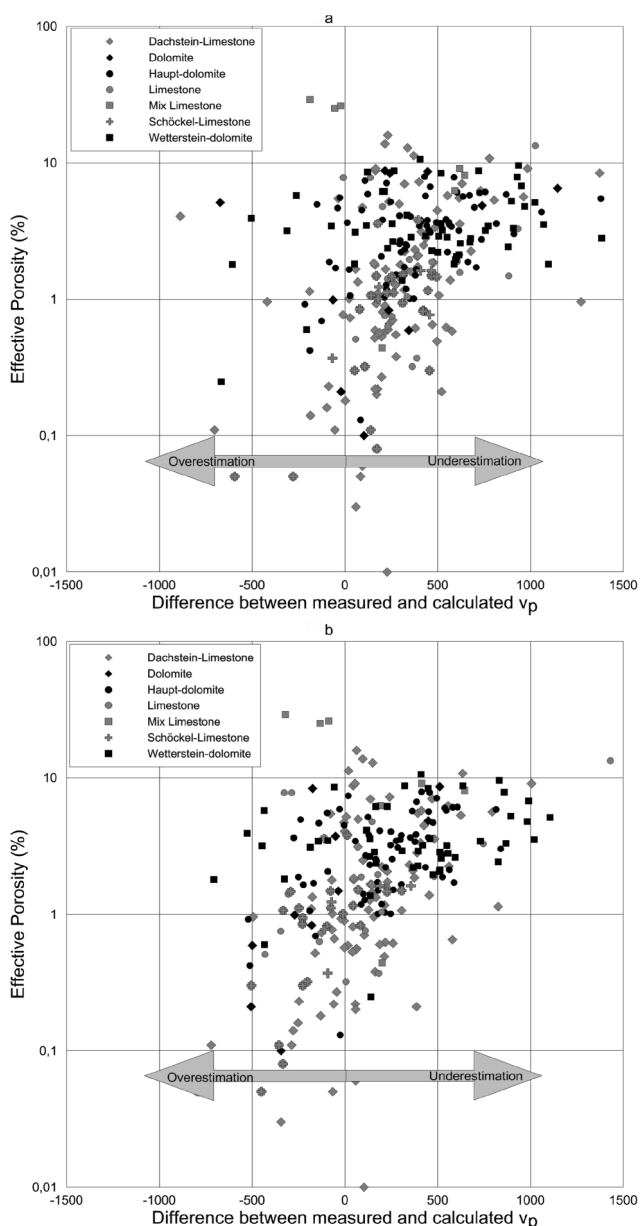
Figure 4: Difference between $v_{p,sat,measured}-v_{p,calculated}$ versus effective porosity, a: Gassmann equation using k_{dry}, b: the two other approaches (k_1 and λ) deliver the same result, grey: limestone samples, black: dolomite samples.
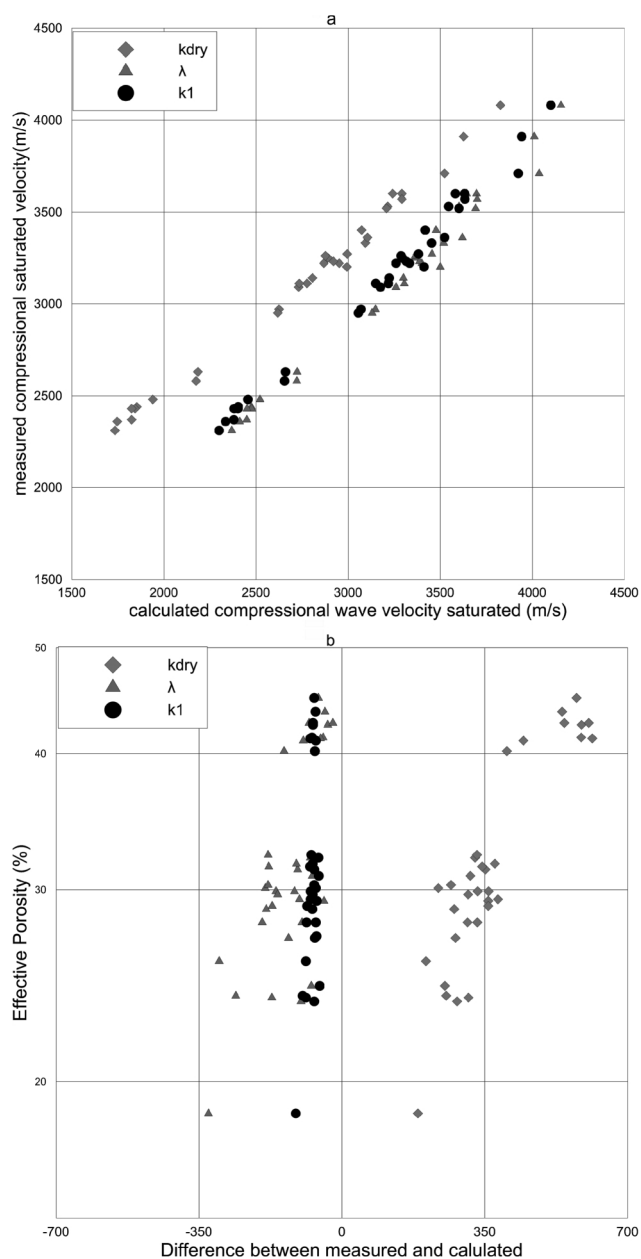
Figure 5: Data set of Røgen et al (2005), a: measured versus calculated v_p: Gassmann with k_{dry} (light grey) and Gassmann using k_1 (black) and using λ (dark grey). b: $v_{p,measured}-v_{p,calculated}$ versus porosity.

the dry rock frame delivers an underestimation of compressional wave velocity for data of saturated samples with high and low porosity. The other two approaches using k_1 and λ directly from the measured data, rather than calculating k_{dry}, delivers the same results for the Austrian carbonates. Results are better than when using k_{dry}, which is what was expected. The results were nearly the same for the samples with high porosities. Using k_{dry} underestimates the velocities. In contrast to the Austrian carbonates, there is a difference between using k_1 and λ, especially for the samples with a lower porosity. The complete data set of the chalk limestone is overestimated by about 100m/s using k_1.

It is thus proved that for laboratory data there is no need to additionally calculate kdry for carbonates, and that an application with λ works too. The data scatter, which can still be observed, may result from the various pore types and the shear weakening effect. Including this in further work, if possible, could result in the optimal result. Additionally, further work will focus on the application and testing of additional log data. The experimentally verified data show that a fluid substitution is also possible for carbonates and can be applied for example

- for discrimination of gas and water in hydrocarbon exploration as well as in monitoring
- for seismic interpretation and modelling in carbonate massive with respect to critical hydrogeological situations

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