ANALYSIS OF DIFFUSIVITY EQUATION USING DIFFERENTIAL QUADRATURE METHOD

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Evaluation of exact analytical solution for flow to a well, under the assumptions made in its development commonly requires large amounts of computation time and can produce inaccurate results for selected combinations of parameters. Large computation times occur because the solution involves the infinite series. Each term of the series requires evaluation of exponentials and Bessel functions, and the series itself is sometimes slowly convergent. Inaccuracies can result from lack of computer precision or from the use of improper methods of numerical computation. This paper presents a computationally efficient and an accurate new methodology in differential quadrature analysis of diffusivity equation to overcome these difficulties. The methodology would overcome the difficulties in boundary conditions implementations of second order partial differential equations encountered in such problems. The weighting coefficients employed are not exclusive, and any accurate and efficient method such as the generalized differential quadrature method may be used to produce the method's weighting coefficients. By solving finite and infinite boundary condition in diffusivity equation and by comparing the results with those of existing solutions and/or those of other methodologies, accuracy, convergences, reduction of computation time, and efficiency of the methodology are asserted.

Key words: Differential quadrature, finite-radial reservoir, infinite-radial reservoir, pseudo-steady state, unsteady-state, diffusivity equation.

1. INTRODUCTION

Numerical methods play a fundamental role in solving the dynamics of real world phenomena, see, for example, Refs. [1–3]. In seeking alternative numerical algorithms, using less grid points with acceptable accurate solutions to differential

equations, another numerical scheme called differential quadrature method (DQM) was introduced in Ref. [4] and was further developed in Ref. [5]. Bert and his coworkers [6–11] contributed to the development of the method and used DQM for the analysis of structural problems.

In the implementation of this method to structural problems one would encounter a difficulty in imposing the boundary conditions. To solve this difficulty several different schemes have been introduced.

There are several methods to evaluate the reservoir parameters [12]. It was showed that solutions to differential equations describing flow in petroleum reservoir for given initial and boundary conditions can be expressed compactly using dimensionless variables and parameters. It was examined several of these solutions that are important in reservoir engineering applications, see Refs. [13–18].

The solution of the diffusivity equation models radial flow of a slightly compressible liquid in a homogeneous reservoir of uniform thickness; reservoir at uniform pressure p_i before production; no flow across the outer boundary (at $r = r_e$); and production at constant rate q from the single well (centered in the reservoir) with wellbore radius r_w [19]. The solution-pressure as a function of time and radius for fixed values of r_e, r_w, and rock and fluid properties is expressed most conveniently in terms of dimensionless variables and parameters $p_D = f(t_D, r_D, r_{eD})$, which states that p_D is a function of the variables t_D and r_D for a fixed value of the parameter r_{eD}. The most important solution is that for pressure at the wellbore radius ($r = r_w$ or $r_D = 1$): $p_D \big|_{r_D=1} = f(t_D, r_{eD})$.

The most useful form of the diffusivity equation solution relates flowing pressure, p_{wf}, at the sandface to time and to reservoir rock and fluid properties. When expressed in terms of dimensionless pressure evaluated at $r_D = 1$, solution shows the functional form of $f(t_D, r_{eD})$, an infinite series of exponentials and Bessel functions [20]. This series has been evaluated for several values of r_{eD} over a wide range of values of t_D [13]. Chatas tabulated these solutions [21]. In order to generalize DQ's application to multi-dimensions and to different conditions and geometry, a new one-dimensional differential quadrature method formulation is presented and implemented.

This manuscript has the following structure: after the introduction, in Section 2, the differential quadrature method is presented. The main problem is stated in Section 3. In Section 4 we impose the boundary conditions for the diffusivity equation for various physical reservoirs. Numerical approach as the main contribution of the paper is presented in Section 5. In Section 6 some numerical examples are illustrated. Finally, the concluding remarks are discussed in Section 7.

2. THE DIFFERENTIAL QUADRATURE METHOD

The DQM is based on the idea that the partial derivative of a field variable at the i-th discrete point in the computational domain is approximated by a weighted linear sum of the field variable along the line that passes through that point, which is parallel to the coordinate direction of the derivative [22]. For example, the mth derivative of the field variable $p(r,t)$ at point r_i is approximated as

$$\frac{\partial^m p}{\partial r^m}\bigg|_{r=r_i} = \sum_{j=1}^{N} A_{ij}^{(m)} p(r_j,t) \tag{1}$$

where $A_{ij}^{(m)}$ are the weighting coefficients associated with the m-th order derivative and N is the number of grid points in the r-direction.

There are two points in successful applications of the DQM; one is how to determine the weighting coefficients and the other one is how to select the grid points. In order to obtain the weighting coefficients, one may use the polynomial test functions in Eq. (1) and solve the Vandermonde system of equations by using the usual linear equation solver. However, the Vandermonde matrices are known to be inherently ill-conditioned, and increasing the number of grid points causes Vandermonde equations to become increasingly inaccurate. Another method for evaluating the weighting coefficients in our analysis is GDQM. In the present method, the weighting coefficients for the derivatives may be obtained directly, irrespective of the number and position of the grid points from an explicit formula [11]. These coefficients for the first-order derivatives are given by

$$A_{ij}^{(1)} = \begin{cases} \dfrac{M^{(1)}(x_i)}{(x_i - x_j)M^{(1)}(x_j)} & i, j = 1, 2, ..., N, \\[2mm] -\displaystyle\sum_{j=1, j\neq i}^{N} A_{ij}^{(1)} & i = j, \ i, j = 1, 2, ..., N, \end{cases} \tag{2}$$

where $M(r)$ and $M^{(1)}(r)$ are defined as

$$M(r_i) = \prod_{j=1}^{N}(r_i - r_j), \quad M^{(1)}(r_i) = \prod_{j=1, j\neq i}^{N}(r_i - r_j) \tag{3}$$

The weighting coefficients of higher-order derivatives are obtained from the following recurrence relationship,

$$A_{ij}^{(m)} = m\left[A_{ii}^{(m-1)} A_{ij}^{(1)} - \frac{A_{ij}^{(m-1)}}{(r_i - r_j)} \right] \tag{4}$$

for $i, j = 1, 2, ..., N, \ i \neq j,$ and $m \geq 2.$

A natural and often convenient choice for the grid points are the equally spaced points. Another choice which gives more accurate results, are unequally spaced grid points [8]. A well accepted set of grid points is the cosine type (or the Gauss–Lobatto–Chebyshev) points given by

$$r_i = \frac{1}{2}\left[1 - \cos\frac{(i-1)\pi}{N-1} \right].$$
(5)

In this work, unequally spaced grid points in conjunction with GDQM are used to evaluate the weighting coefficients [23, 24].

3. THE PROBLEM STATEMENT

To develop diffusivity equation some assumptions are assumed. These assumptions are introduced as needed, to combine (1) the law of conservation of mass, (2) Darcy's law, and (3) equations of state to achieve our objectives [19–20], [25–26].

Consider radial flow toward a well in a circular reservoir. Combining the law of conservation of mass and Darcy's law for the isothermal flow of fluids of small and constant compressibility, a partial differential equation is obtained that simplifies to

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi\mu\, c}{0.000264\ k}\frac{\partial p}{\partial t}$$
(6)

where it is assumed that compressibility, $c,$ is small and independent of pressure; permeability, $k,$ is constant and isotropic; viscosity, $\mu,$ is independent of pressure; porosity, $\phi,$ is constant; and that certain terms in the basic differential equation (involving pressure gradients squared) are negligible. This equation is called the diffusivity equation. The term $0.000264k/\mu\phi c$ is called the hydraulic diffusivity and frequently is given by the symbol $\eta.$

It is convenient and customary to present graphical or tabulated solutions to flow equations. Such as Eq. (6), in terms of dimensionless variables. In this way, it is possible to present compact solutions for a wide range of parameters $\phi, \mu, c,$ and $k,$ and variables $r, p,$ and $t.$

It should be identified the dimensionless variables and parameters required to characterize the solutions to the equations describing radial flow of a slightly compressible liquid in a reservoir. It is assumed that Eq. (6) adequately models this flow. Specifically, we analyze the situation in which (1) pressure throughout the

reservoir is uniform before production; (2) fluid is produced at a constant rate from a single well of radius r_w centered in the reservoir; and (3) there is no flow across the outer boundary (with radius r_e) of the reservoir. Stated mathematically, the differential equation, and initial and boundary conditions are

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial p}{\partial r}) = \frac{\phi\mu c}{0.000264\ k}\frac{\partial p}{\partial t},$$ (7)

at $t = 0,\ p = p_i$ for all r,

at $r = r_e,\quad q = 0$ for $t > 0$, or $\left.\frac{\partial p}{\partial r}\right|_{r_e} = 0,$

at $r = r_w,\ q = \frac{-0.001127(2\pi r_w h)}{B}\frac{k}{\mu}\frac{\partial p}{\partial r}$ for $t > 0$ or $\left.\frac{\partial p}{\partial r}\right|_{r_w} = -\frac{qB\mu}{0.00708\ khr_w}$

It may be defined a dimensionless radius, $r_D = \frac{r}{r_w}$ (any other convenient reference length, such as r_e, could have been used). From the form of the differential equation, a convenient definition of dimensionless time is $t_D = \frac{0.000264\ kt}{\phi\mu c\ r_w^2}.$

The initial and boundary conditions suggest that a convenient definition of dimensionless pressure is $p_D = \frac{0.00708\ kh\ (p_i - p)}{qB\mu}.$

With this definition, the boundary condition of Eq. (7) becomes

$\frac{-qB\mu}{0.00708}\left.\frac{\partial p_D}{\partial r_D}\right|_{r_D=1} = \frac{-qB\mu}{0.00708\ khr_w}$, or simply $\left.\frac{\partial p_D}{\partial r_D}\right|_{r_D=1} = 1$.

Expressed in terms of dimensionless variables, the differential equation and its initial and boundary conditions become

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}(r_D\frac{\partial p_D}{\partial r_D}) = \frac{\partial p_D}{\partial t_D},$$ (8)

or

$$\frac{1}{r_D}\frac{\partial p_D}{\partial r_D} + \frac{\partial^2 p_D}{\partial r_D^2} = \frac{\partial p_D}{\partial t_D}$$ (9)

$p_D = 0$ for all r_D at $t_D = 0,$ $\left.\frac{\partial p_D}{\partial r_D}\right|_{r_D = \frac{r_e}{r_w} = r_{De}} = 0$ for $t_D > 0,$ $\left.\frac{\partial p_D}{\partial r_D}\right|_{r_D=1} = 1$ for $t_D > 0$.

Dimensionless variables are

$$r_D = \frac{r}{r_w}, \ t_D = \frac{0.000264 \ kt}{\phi\mu c r_w^2}, \text{ and } p_D = \frac{0.00708 \ kh(p_i - p)}{qB\mu}.$$

Obviously, r_D, is dimensionless. It is easy to show that t_D and p_D are dimensionless, too [27].

4. THE BOUNDARY CONDITIONS

There are four solutions to Eq. (6) that are particularly useful in well testing: the solution for a bounded cylindrical reservoir; the solution for an infinite reservoir with a well, considered to be a line source with zero wellbore radius; the pseudosteady-state solution; and the solution that includes wellbore storage for a well in an infinite reservoir. Below we summarize the assumptions that were necessary to develop Eq. (6), namely: homogeneous and isotropic porous medium of uniform thickness; pressure-independent rock and fluid properties; small pressure gradients; radial flow; applicability of Darcy's law (sometimes called laminar flow); and negligible gravity forces. We will introduce further assumptions to obtain solutions of Eq. (6).

4.1. BOUNDED CYLINDRICAL RESERVOIR

Solution of Eq. (6) requires specifying two boundary conditions and an initial condition. A realistic and practical solution is obtained if we assume that (1) a well produces a constant rate, qB, into the wellbore (q refers to flow rate in STB/D at surface conditions, and B is the formation volume factor in RB/STB); (2) the well, with wellbore radius r_w, is centered in a cylindrical reservoir of radius r_e, and that there is no flow across this outer boundary; and (3) before production begins, the reservoir is at uniform pressure, p_i. The most useful form of the desired solution relates flowing pressure, p_{wf}, at the sandface to time and to reservoir rock and fluid properties.

At $t = 0$, $p = p_i$ for all r,

at $r = r_e$, $q = 0$ for $t > 0$, or $\frac{\partial p}{\partial r}\big|_{r_e} = 0$,

at $r = r_w$, $q = \frac{-0.001127(2\pi r_w h)}{B}\frac{k}{\mu}\frac{\partial p}{\partial r}$ for $t > 0$ or $\frac{\partial p}{\partial r}\big|_{r_w} = -\frac{qB\mu}{0.00708 \ khr_w}$

or $p_D = 0$ for all r_D at $t_D = 0$, $\frac{\partial p_D}{\partial r_D}\big|_{r_D = r_e/r_w = r_{De}} = 0$ for $t_D > 0$, $\frac{\partial p_D}{\partial r_D}\big|_{r_D=1} = 1$ for $t_D > 0$.

These initial and boundary conditions were for finite-radial reservoir [27].

4.2. INFINITE CYLINDRICAL RESERVOIR WITH LINE-SOURCE WELL

Assume that (1) a well produces at a constant rate, qB; (2) the well has zero radius; (3) the reservoir is at uniform pressure, p_i, before production begins; and (4) the well drains an infinite area (*i.e.*, that $p \to p_i$ as $r \to \infty$).

at $t = 0$, $p = p_i$ for all r,

at $p = p_i$ for $t > 0$,

at $r = r_w$, $q = \dfrac{-0.001127(2\pi r_w h)}{B} \dfrac{k}{\mu} \dfrac{\partial p}{\partial r}$ for $t > 0$

or

$p_D = 0$ for all r_D at $t_D = 0$,

$p_D = 0$ for $t_D > 0$ at $r_D = r_{De}$

$\dfrac{\partial p_D}{\partial r_D} |_{r_D=1} = 1$, for $t_D > 0$,

These initial and boundary conditions were for infinite-radial reservoir [27].

5. THE NUMERICAL APPROACH

Applying differential quadrature method on diffusivity equation gives

$$\frac{1}{r_{Di}} \sum_{j=1}^{N} A_{ij}^{(1)} p_{Dj} + \sum_{j=1}^{N} A_{ij}^{(2)} p_{Dj} = \frac{\partial p_{Di}}{\partial t_D} \tag{10}$$

For $i = 2, ..., N-1$.

The boundary conditions of the finite reservoir can be written as

$$\sum_{j=1}^{N} A_{1j}^{(1)} p_{Dj} = 1 \qquad \text{for} \quad t_D > 0, \tag{11}$$

$$\sum_{j=1}^{N} A_{Nj}^{(1)} p_{Dj} = 0 \qquad \text{for} \quad t_D > 0, \tag{12}$$

Solving Eqs. (11) and (12) gives

$$p_{D1} = \frac{A_{NN}^{(1)} + \sum_{j=2}^{N-1}(A_{1N}^{(1)}A_{Nj}^{(1)} - A_{NN}^{(1)}A_{1j}^{(1)})p_{Dj}}{A_{11}^{(1)}A_{NN}^{(1)} - A_{1N}^{(1)}A_{N1}^{(1)}} \tag{13}$$

$$p_{DN} = \frac{-A_{N1}^{(1)} + \sum_{j=2}^{N-1}(A_{N1}^{(1)}A_{1j}^{(1)} - A_{11}^{(1)}A_{Nj}^{(1)})p_{Dj}}{A_{11}^{(1)}A_{NN}^{(1)} - A_{1N}^{(1)}A_{N1}^{(1)}} \tag{14}$$

For infinite reservoir, for the purposes of convenience during the numerical computations, r_D is normalized by

$$R_D = 1 - \exp(1 - r_D) \tag{15}$$

Using Eq. (15), Eq. (9) can be transformed to,

$$(1 - R_D)\frac{\ln(1 - R_D)}{1 - \ln(1 - R_D)}\frac{\partial p_D}{\partial R_D} + (1 - R_D)^2\frac{\partial^2 p_D}{\partial R_D^2} = \frac{\partial p_D}{\partial t_D} \tag{16}$$

and DQ discretization is

$$\left((1 - R_{Di})\frac{\ln(1 - R_{Di})}{1 - \ln(1 - R_{Di})}\right)\sum_{j=1}^{N}A_{ij}^{(1)}p_{Dj} + (1 - R_{Di})^2\sum_{j=1}^{N}A_{ij}^{(2)}p_{Dj} = \frac{\partial p_{Di}}{\partial t_D} \tag{17}$$

For infinite reservoir, DQM gives the following boundary conditions

$$\sum_{j=1}^{N}A_{1j}^{(1)}p_{Dj} = 1 \quad \text{for} \quad t_D > 0, \tag{18}$$

$$p_{DN} = 0 \quad \text{for} \quad t_D > 0 \tag{19}$$

Solving Eqs. (18) and (19) yields

$$p_{D1} = \frac{1 - \sum_{j=2}^{N-1}A_{1j}^{(1)}p_{Dj}}{A_{11}^{(1)}} \tag{20}$$

Substituting Eqs. (13) and (14) into Eq. (10), a system of linear equations is obtained that may be solved by any standard method, namely

$$\sum_{j=2}^{N-1}(\frac{1}{r_{Di}}\frac{A_{i1}^{(1)}}{A_{11}^{(1)}A_{NN}^{(1)}-A_{1N}^{(1)}A_{N1}^{(1)}}(A_{1N}^{(1)}A_{Nj}^{(1)}-A_{NN}^{(1)}A_{1j}^{(1)})+\frac{1}{r_{Di}}A_{ij}^{(1)}+$$

$$+\frac{1}{r_{Di}}\frac{A_{iN}^{(1)}}{A_{11}^{(1)}A_{NN}^{(1)}-A_{1N}^{(1)}A_{N1}^{(1)}}(A_{N1}^{(1)}A_{1j}^{(1)}-A_{11}^{(1)}A_{Nj}^{(1)})+$$

$$+\frac{A_{i1}^{(2)}}{A_{11}^{(1)}A_{NN}^{(1)}-A_{1N}^{(1)}A_{N1}^{(1)}}(A_{1N}^{(1)}A_{Nj}^{(1)}-A_{NN}^{(1)}A_{1j}^{(1)})+A_{ij}^{(2)}+$$

$$+\frac{A_{iN}^{(2)}}{A_{11}^{(1)}A_{NN}^{(1)}-A_{1N}^{(1)}A_{N1}^{(1)}}(A_{N1}^{(1)}A_{1j}^{(1)}-A_{11}^{(1)}A_{Nj}^{(1)}))p_{Dj}+\frac{1}{r_{Di}}\frac{A_{i1}^{(1)}A_{NN}^{(1)}}{A_{11}^{(1)}A_{NN}^{(1)}-A_{1N}^{(1)}A_{N1}^{(1)}}+ \tag{21}$$

$$-\frac{1}{r_{Di}}\frac{A_{iN}^{(1)}A_{N1}^{(1)}}{A_{11}^{(1)}A_{NN}^{(1)}-A_{1N}^{(1)}A_{N1}^{(1)}}+$$

$$+\frac{A_{i1}^{(2)}A_{NN}^{(1)}}{A_{11}^{(1)}A_{NN}^{(1)}-A_{1N}^{(1)}A_{N1}^{(1)}}-\frac{A_{iN}^{(2)}A_{N1}^{(1)}}{A_{11}^{(1)}A_{NN}^{(1)}-A_{1N}^{(1)}A_{N1}^{(1)}}=\frac{\partial p_{Di}}{\partial t}$$

for $i=2,\ 3,...,\ N-1$, for finite reservoir.

Substituting Eqs. (19) and (20) in Eq. (16), a system of linear equations is resulted that can be solved by any standard method

$$\sum_{j=2}^{N-1}(-\frac{(1-R_{Di})\ln(1-R_{Di})}{1-\ln(1-R_{Di})}\frac{A_{i1}^{(1)}A_{1j}^{(1)}}{A_{11}^{(1)}}+\frac{(1-R_{Di})\ln(1-R_{Di})}{1-\ln(1-R_{Di})}A_{ij}^{(1)}$$

$$-(1-R_{Di})^2\frac{A_{i1}^{(2)}A_{1j}^{(1)}}{A_{11}^{(1)}}+(1-R_{Di})^2 A_{ij}^{(2)})p_{Dj}+ \tag{22}$$

$$+\frac{(1-R_{Di})\ln(1-R_{Di})}{1-\ln(1-R_{Di})}\frac{A_{i1}^{(1)}}{A_{11}^{(1)}}+(1-R_{Di})^2\frac{A_{i1}^{(2)}}{A_{11}^{(1)}}=\frac{\partial p_{Di}}{\partial t_{D}}$$

for $i=2,3,...,N-1$, for infinite reservoir.

The final results are the initial value problems. These equations can be solved by some standard methods such as fourth-order Runge-Kutta method. The first case is the result of finite reservoir and the last case is the infinite reservoir result.

In the above DQ discretization the grid points are the cosine type (or the Gauss–Lobatto–Chebyshev) points given by

$$r_{Di}=\frac{1}{2}(1-\cos(\frac{i-1}{N-1})\pi)(r_{eD}-1)+1 \tag{23}$$

for finite reservoir and

$$R_{Di}=\frac{1}{2}(1-\cos(\frac{i-1}{N-1})\pi) \tag{24}$$

for infinite reservoir.

6. NUMERICAL EXAMPLES

To illustrate the generality and accuracy of the present DQM, this procedure is applied to diffusivity equation problem under various boundary conditions. The problem under boundary conditions was considered by other researchers [28–34]. The results are compared with those of the same problems solved by van Everdingen and Hurst [13].

Tables 1 and 2 show the computation times of DQM, which are less than one second in all cases. The computation times for finite difference method are considerably more than DQM. However, the computation time for finite difference method is not considered in the presented method (DQM).

Figure 1 shows the analysis results for infinite reservoirs. Figures 2, 3, 4 and 5 are the results of finite reservoirs for r_{eD} = 1.5, 4, 7, and 10, respectively. As it can be shown the number of grid points has no effect on the infinite acting reservoir and finite reservoir up to r_{eD} = 7. However, higher t_D in the case of infinite reservoir needs more number of grid points. In the case of finite reservoir more number of grid points resulted in a better prediction.

As it can be seen from Figures 1, 2, 3, 4, and 5, the stability of the DQM is independent of number of grid points. So, this is a good property of this method. The other necessary condition for convergence is consistency that it is very good and it can be visualized from the following figures. However, the consistency is better for lower r_{eD}. For the greater number of grid points, better results can be reported.

Table 1

Computation of time for infinite-radial system

t_D	Computational time (Second)		
	N=9	*N=11*	*N=15*
0.003	0.001769	0.003821	0.004268
0.008	0.002255	0.002263	0.004096
0.007	0.002769	0.003660	0.005187
0.2	0.002663	0.002084	0.004913
0.9	0.002763	0.001926	0.004056
1.4	0.001461	0.002910	0.005157
6.0	0.002680	0.001918	0.003649
10.0	0.001379	0.003742	0.004652
40.0	0.001389	0.001941	0.004408

Table 2

Computation of time for finite-radial system with closed exterior boundary

| r_{eD}	t_D	*Computational time (Second)*		
		N=9	*N=11*	*N=15*
1.5	0.06	0.001025	0.001520	0.002690
	0.24	0.001977	0.001616	0.003233
	0.45	0.001217	0.001678	0.003285
4	1.5	0.001226	0.001513	0.002900
	2.8	0.001165	0.002865	0.003412
	5.5	0.001236	0.001517	0.002703
7	6.0	0.001159	0.001565	0.002619
	11.0	0.001106	0.001796	0.003315
	17.0	0.000975	0.001400	0.002323
10	12.0	0.001117	0.001520	0.002391
	17.0	0.001137	0.001515	0.002913
	26.0	0.001119	0.001565	0.003280
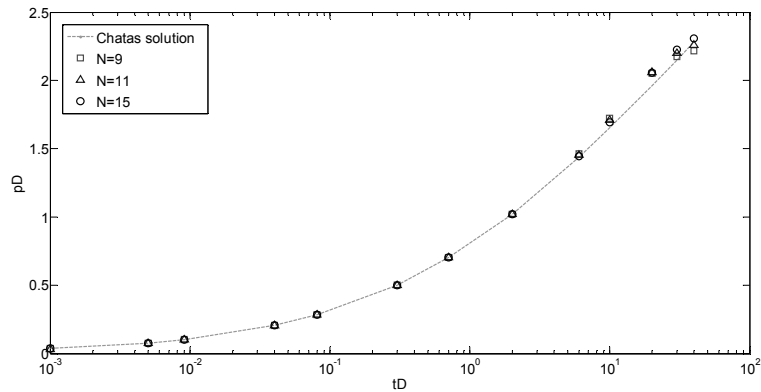
Fig. 1 – Plot of p_D *vs.* t_D for infinite acting reservoir.
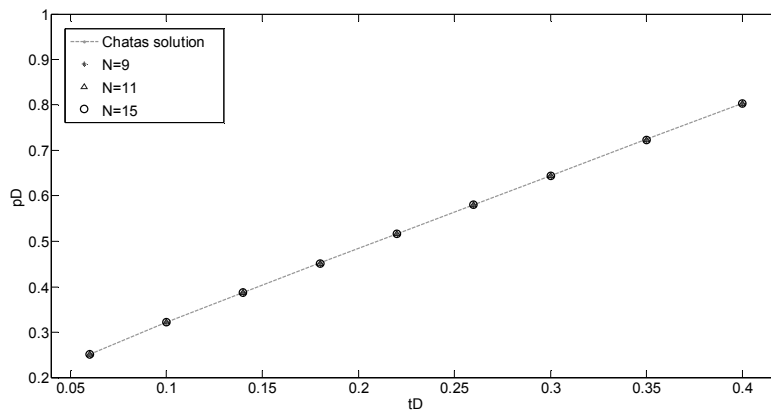

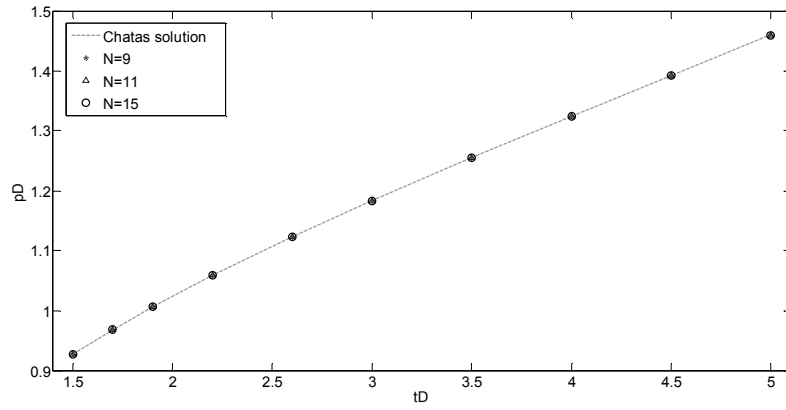Fig. 2 – Plot of p_D *vs.* t_D for finite reservoir where r_{eD}=1.5.

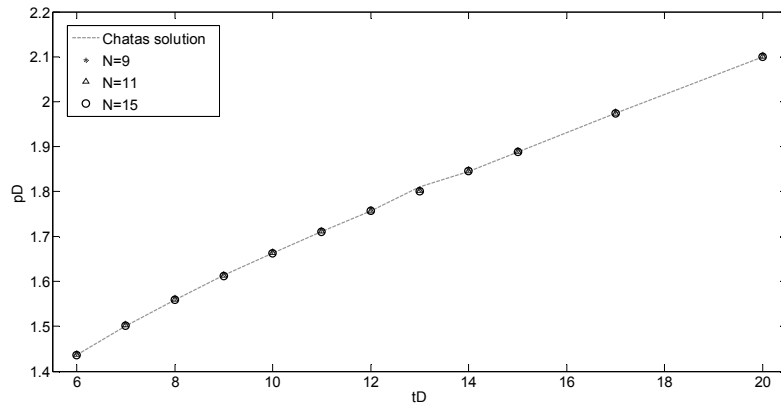Fig. 3 – Plot of p_D *vs.* t_D for finite reservoir where r_{eD}=4.

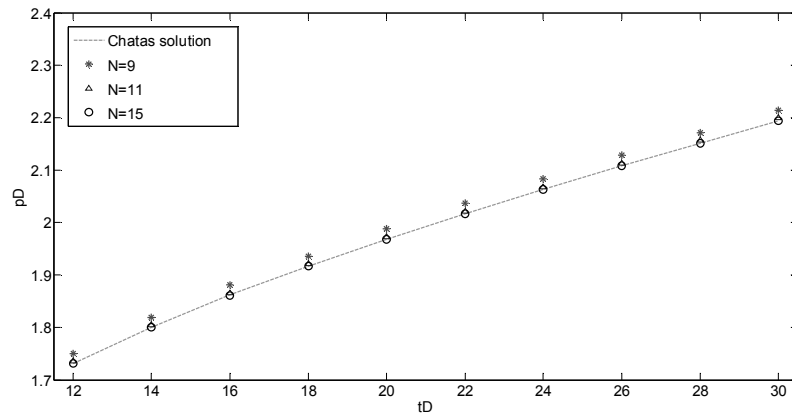Fig. 4 – Plot of p_D *vs.* t_D for finite reservoir where r_{eD}=7.

Fig. 5 – Plot of p_D *vs.* t_D for finite reservoir where r_{eD}=10.

7. CONCLUSIONS

A new one-dimensional DQM was introduced to show the applicability of the proposed DQM to one dimensional radial problem with two forms of geometry or boundary conditions. Examples have been presented for diffusivity equation analysis. Fast rate of convergence is demonstrated and the results are in excellent agreement with the solutions of other methods even with few numbers of DQ grid points in all examples presented. Acceptable results with a few number of grid points can be obtained and there is no concern about instability problems due to selection of large time steps. In other words, DQM can be claimed as an unconditionally stable and efficient method for numerical simulation of porous media. The results obtained confirm the applicability of the present DQ methodology and can be used for solving a variety of diffusivity equation problems. Due to the high accuracy of the presented approach, these solutions can be used as benchmark for future works.

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